



Progression in Bar Modelling

Primary Maths

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KEY MODELS

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Introduction

Bar modelling is used widely in Singapore, Japan and USA as an effective part of the **Concrete, Pictorial, Abstract (CPA)** approach to the mastery of mathematics. Concrete materials are embedded alongside pictorial representations and abstract expressions to ensure procedural fluency and conceptual understanding are developed in tandem. It is not a method for problem solving but does reveal the mathematical structure beneath the problem and mathematical relationships between its component parts. Many teachers have found the approach very effective in gaining an enduring understanding for children by providing many tangible and memorable experiences to draw upon in their learning.

National Curriculum

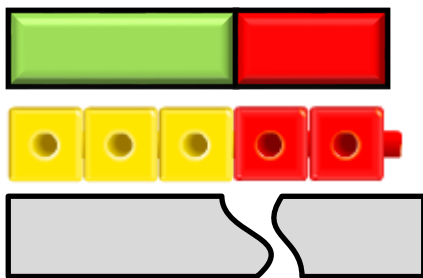
As we know, the aims of the national curriculum require the children to:

- become **fluent** ... through varied and frequent practice with increasingly complex problems [to] **develop conceptual understanding**
- **reason mathematically**
- **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication.

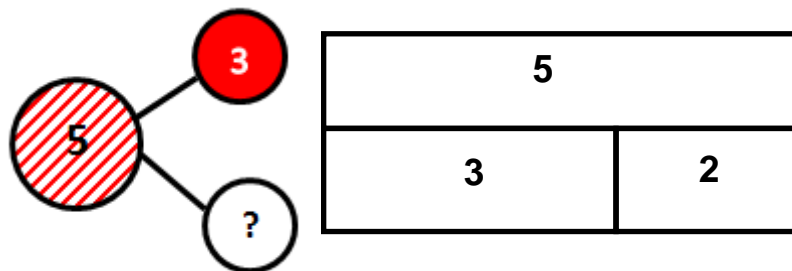
In the Herts for Learning document 'Working Mathematically', the implications of these aims are thoroughly explored through **Application, Reasoning** and **Problem Solving Strategies**. When embedded in a rich and varied **CPA** approach, children are provided with ample opportunities to develop and demonstrate skills across the spectrum of the procedural fluency and conceptual understanding required.

The Basics

CONCRETE



PICTORIAL



ABSTRACT

I know that 2 and three more is equal to 5.

$$5 = 3 + 2$$

KEY MODELS

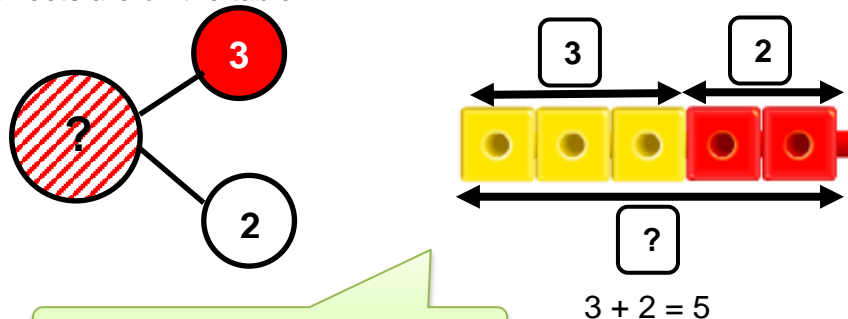
Addition and subtraction

Problems involving addition and subtraction have three possible unknowns. When given the value of two unknowns, the third can be found.

Addition models – Total unknown

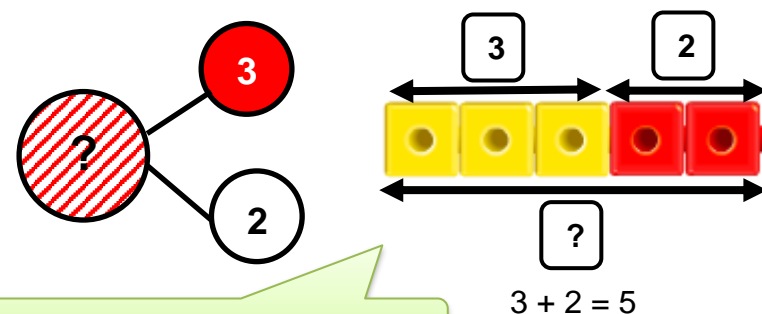
Aggregation (Two quantities (parts) combined)

Three yellow sweets and two red sweets are on the table. How many sweets are on the table?



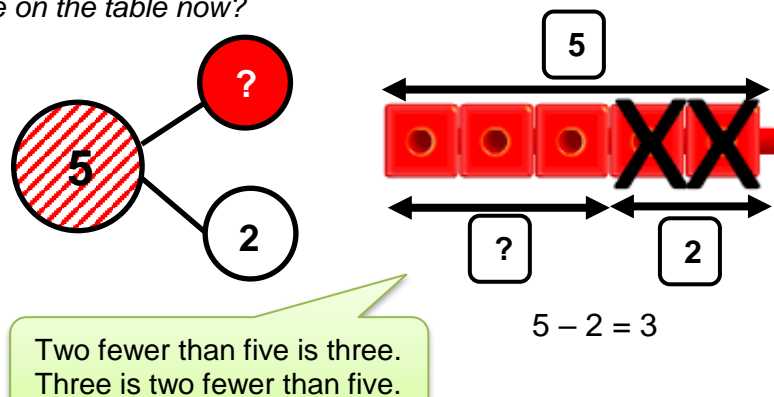
Augmentation (One quantity is increased by a provided amount)

James had three stickers. He was given two more. How many stickers does he have now?

**Subtraction – One of the quantities (parts) is unknown**

Take away (One quantity is decreased by a provided amount)

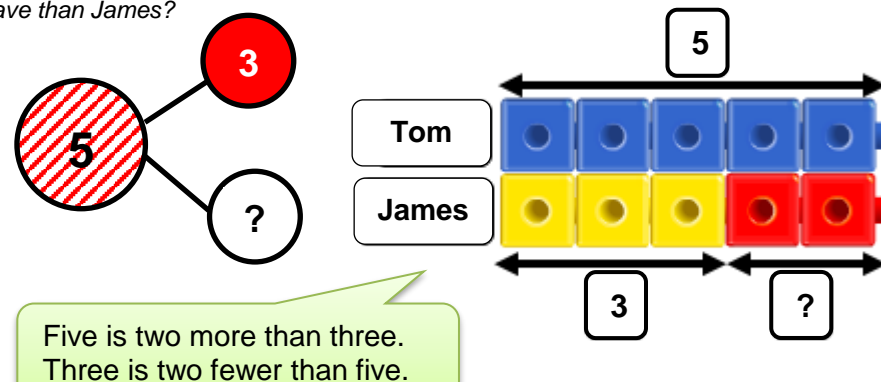
Five sweets were on the table. Tom ate two sweets. How many sweets are on the table now?



Comparison (Find the difference)

*Key model showing the relationship between addition and subtraction

Tom has five sweets and James has three sweets. How many more sweets does Tom have than James?

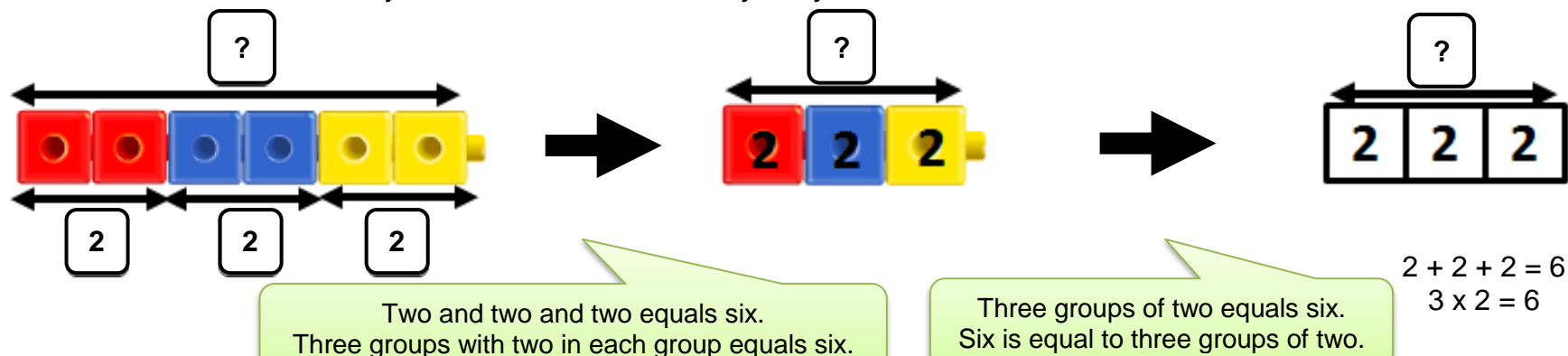


Multiplication and Division: EQUAL GROUPS

Problems involving multiplication and division have three possible unknowns – product, group size or number of groups. When given the value of two unknowns the third can be found. In bar modelling, all blocks represent the replication of an equal unit.

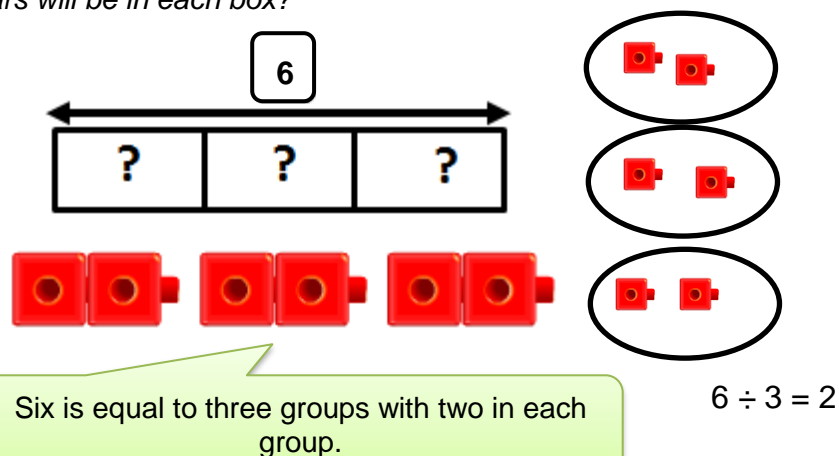
Multiplication - Group size and number of groups provided, product is unknown.

There are three boxes with two teddy bears in each box. How many teddy bears are in there is all?

**Division**

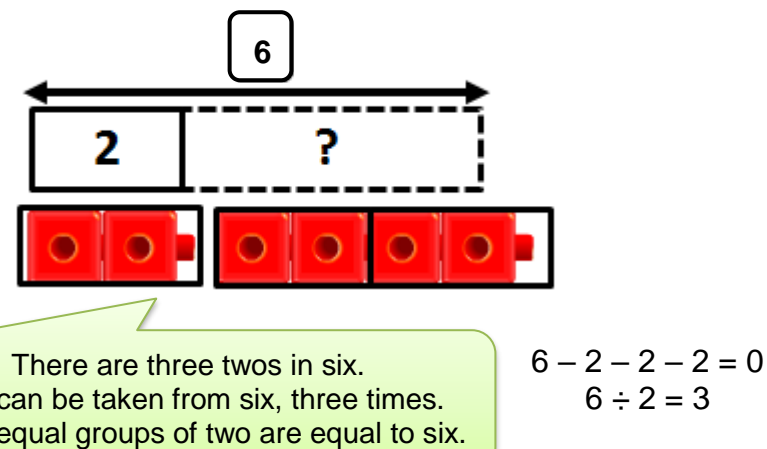
Sharing – Product and number of groups is known, group size is unknown.

If six teddy bears are shared equally into 3 boxes, how many teddy bears will be in each box?



Grouping – Product and groups size is known, number of groups is unknown.

If six teddy bears are to be packed 2 to a box, how many boxes are needed?

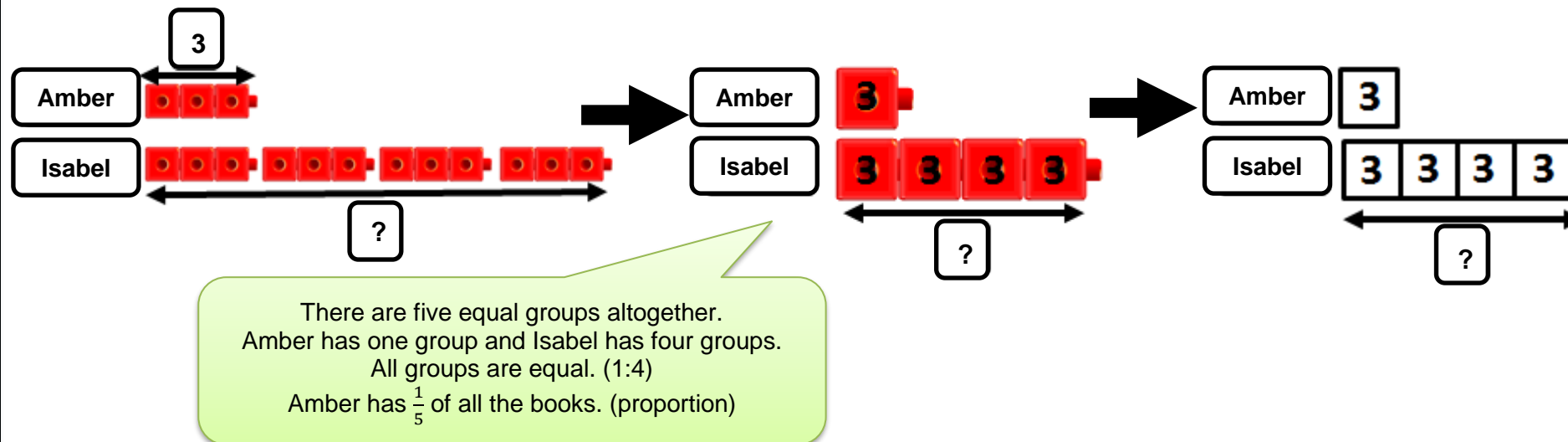


Multiplication and Division: COMPARISON

Problems involving multiplicative comparison occur when two or more sets are compared by showing one set is a number of times larger or smaller than the other set(s). Bar modelling focuses on the multiplicative relationship between the sets.

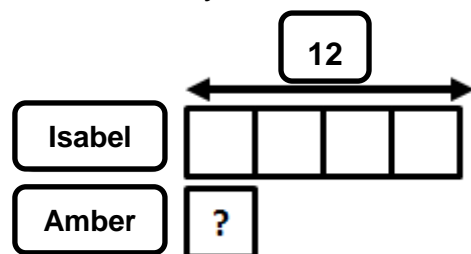
Multiplication – the smaller set and the multiplicative relationship is known, the product is unknown.

Amber read three books. Isabel read four times as many books as Amber. How many books did Isabel read?

**Division**

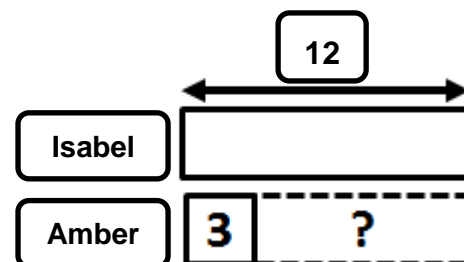
Sharing – the larger set and the multiplicative relationship is known, the smaller set is unknown.

Isabel read 12 books and that is four times as many books as Amber read. How many books did Amber read?



Grouping – the larger set and the smaller set is known, the multiplicative relationship is unknown.

Isabel read 12 books and Amber read 3 books. How many times as many books did Isabel read as Amber did?

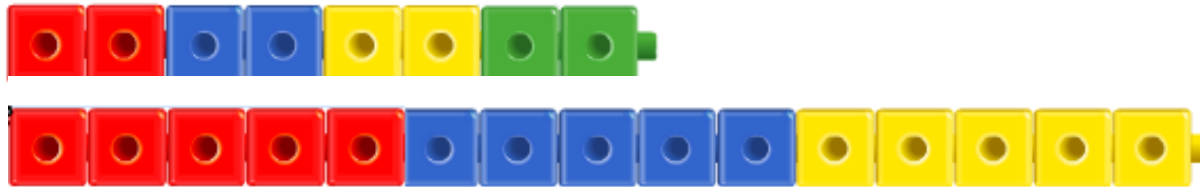


YEAR 1

At this stage, the children will be using multilinking cubes and Cuisenaire rods alongside their own pictorial representations (including a drawn bar model) which are then translated into abstract calculations and language.

Number and place value

- Count in multiples of twos, fives and tens



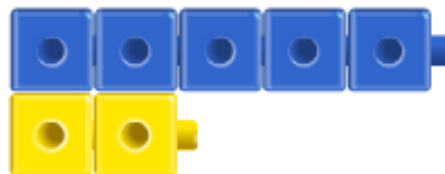
- Given a number, identifies one more and one less
- Identifies and represent numbers using objects and pictorial representations, including the number line and use the language of: equal to, more than, less than (fewer), most, least



One more than seven is eight.
Eight is one more than seven.
Seven is one fewer than eight.



One fewer than seven is six.
Seven is one more than six.
Six is one fewer than seven.



There are more blue cubes than yellow cubes.
There are fewer yellow cubes than blue cubes.
They are not equal.

Addition and Subtraction

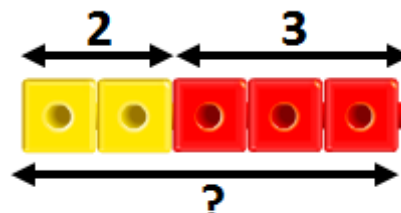
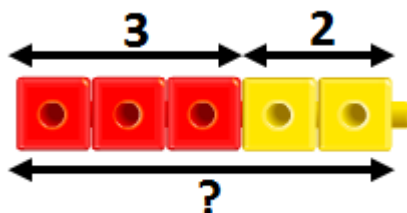
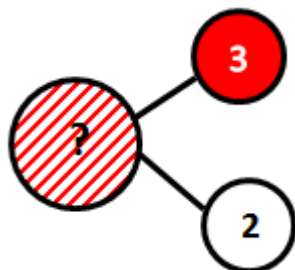
At this stage, the children should be exposed to the different models of addition and subtraction. It is important that they develop deep understanding of how addition and subtraction are related to each other – focus on language and how one model can be interpreted in many different ways.

- Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs
- Represent and use number bonds and related subtraction facts within 20
- Add and subtract one-digit and two-digit numbers to 20, including zero
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$

Addition

Aggregation (Two quantities combined)

Three yellow sweets and two red sweets are on the table. How many sweets are on the table?

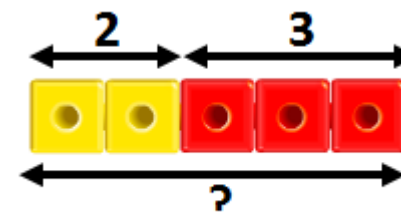
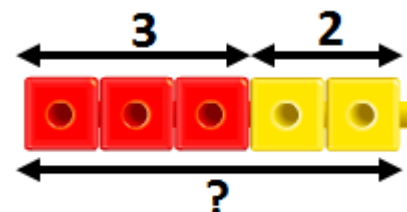
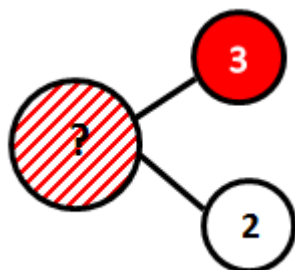


$$\begin{aligned} 3 + 2 &= 5 \\ 5 &= 2 + 3 \\ 2 + 3 &= 5 \\ 5 &= 3 + 2 \end{aligned}$$

Three and two more is equal to five.
Three add two equals five.
Two and three more is equal to five.
Two add three equals five.

Augmentation (one quantity is increased)

Amber had three stickers. She was given two more. How many stickers does she have now?



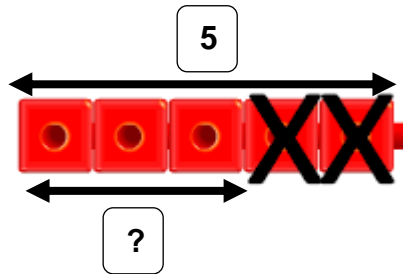
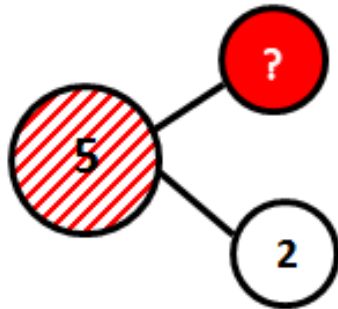
$$\begin{aligned} 3 + 2 &= 5 \\ 5 &= 2 + 3 \\ 2 + 3 &= 5 \\ 5 &= 3 + 2 \end{aligned}$$

Three and two more make five.
Three add two equals five.
Two and three more makes five.
Two add three equals five.

Subtraction

Take away (One quantity is decreased by a provided amount)

Five sweets were on the table. Tom ate two sweets. How many sweets are on the table now?



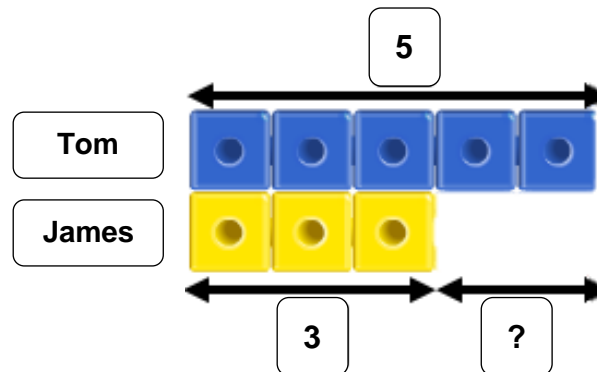
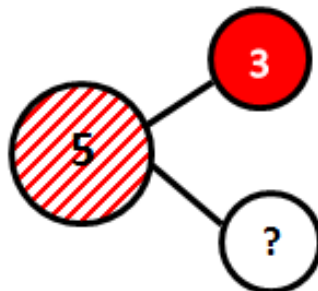
$$5 - 2 = 3$$

$$3 = 5 - 2$$

Two fewer than five is three.
Three is two fewer than five.

Comparison (Find the difference) *Key model showing the relationship between addition and subtraction

Tom has five sweets and James has three sweets. How many more sweets does Tom have than James?



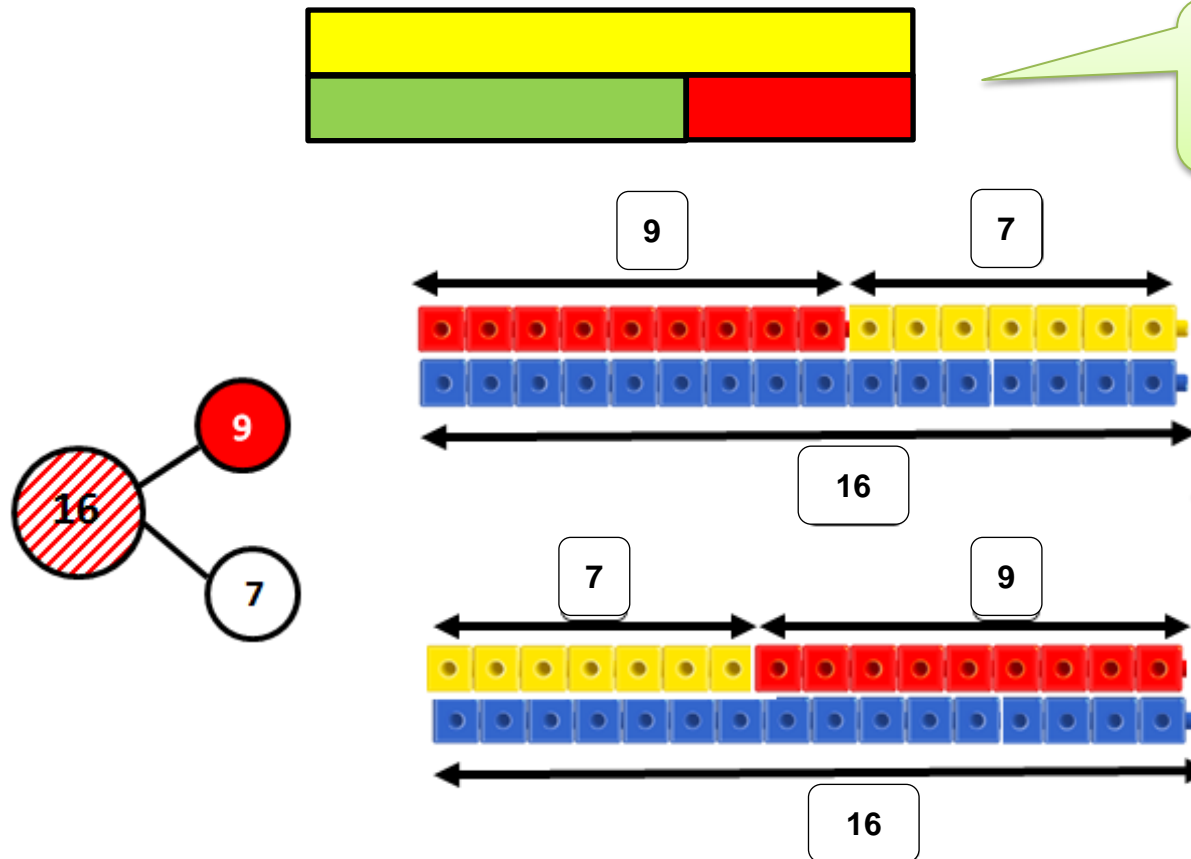
$$5 - 3 = 2$$

$$2 = 5 - 3$$

Five is two more than three.
Three is two fewer than five.

Further examples of the comparison model for Year 1: exploring equality, more than, less/fewer than and also discovering commutativity of addition.

Cuisenaire rods can be used to explore the generalised relationship of addition and subtraction.



Yellow is green add red.
Red add green equals yellow.
Yellow subtract green equals red.
Yellow subtract red equals green.

Seven more than nine is 16.
Seven add nine is equal to 16.
Nine add seven is equal to 16.
Nine more than seven is 16.
Sixteen subtract seven is nine.
Seven fewer than 16 is nine.
Sixteen subtract nine is seven.
Nine fewer than 16 is seven.

$$\begin{aligned} 9 + 7 &= 16 \\ 7 + 9 &= 16 \\ 16 - 9 &= 7 \\ 16 - 7 &= 9 \\ 7 &= 16 - 9 \\ 9 &= 16 - 7 \end{aligned}$$

Ensure that the children explore the models and are able to move the position of the unknown quantity.

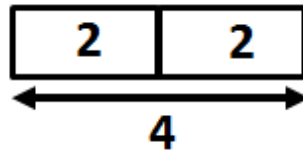
- How would I find the number of red cubes?
- Can you tell me a story problem which makes the seven the unknown?
- Can you tell me about the model using "greater than"?
- What else can you tell me?

Multiplication and Division

At this stage it is important that the children begin to understand that multiplication and division involve the replication of a single unit a number of times and start to explore unitisation e.g. one block is worth two.

- Solve one-step problems involving multiplication and division, by calculating the answer by using concrete objects, pictorial representations and arrays with the support of the teacher

Doubling and halving

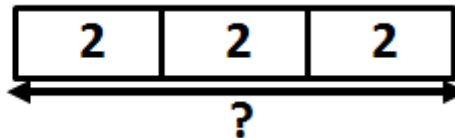


Two and two makes four.
Double two is equal to four.
Half of four is two.

$$\begin{aligned} 2 + 2 &= 4 \\ 2 \times 2 &= 4 \\ 4 - 2 - 2 &= 0 \\ 4 \div 2 &= 2 \end{aligned}$$

Multiplication – Equal groups

Amber wants to give her three friends two lollies each. How many lollies does she need?

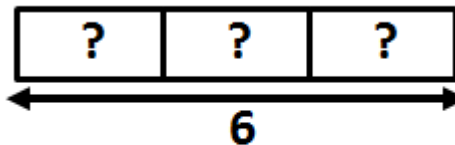


Two and two and two makes six.
Three groups of two make six.

$$\begin{aligned} 2 + 2 + 2 &= 6 \\ 3 \times 2 &= 6 \end{aligned}$$

Division – Sharing

Amber has six lollies. She wants to share them equally between her three friends. How many lollies does each of her friends get?

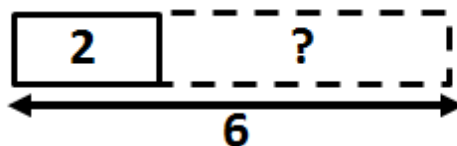


Six shared into three equal groups is two in each group.

$$6 \div 3 = 2$$

Division – Grouping

Amber has six lollies. She wants to give each of her friends two lollies. How many friends can she give lollies to?



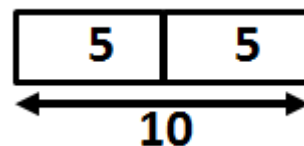
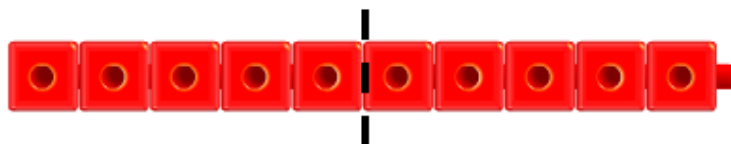
$$6 \div 2 = 3$$

Six grouped into twos equals three

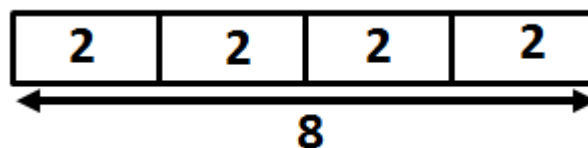
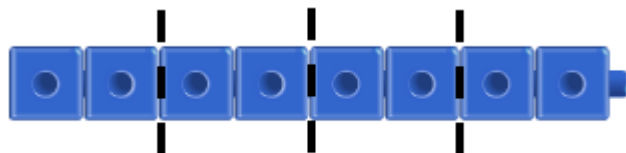
Fractions

- Recognise, find and name a half as one of two equal parts of an object, shape or quantity
- Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity

Cuisenaire rods can be used to explore generalisation in fractions.



Orange is double the yellow.
Yellow is half of the orange.
Yellow add yellow equals orange.
Orange subtract yellow equals orange.



Four reds are equal to brown.
A quarter of brown is one red.
Red add red add red add red equals brown.

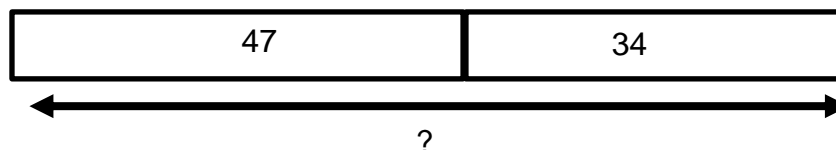
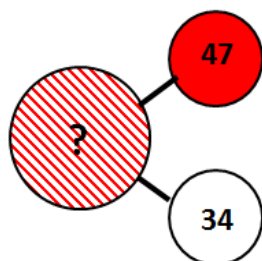
YEAR 2

Addition and subtraction

- Solve problems with addition and subtraction:
 - Using concrete objects and pictorial representations, including those involving numbers, quantities and measures
 - Applying their increasing knowledge of mental and written methods
- Recall and use addition and subtraction facts up to 20 fluently, and derive and use related facts up to 100
- Add and subtract numbers using concrete objects, pictorial representations and mentally
- Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

Addition (Aggregation)

Amber has 47 pence and Isabel has 34 pence. How much do they have altogether?

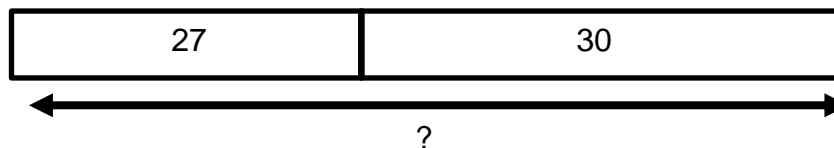
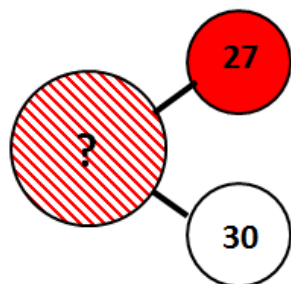


$$\begin{aligned} 81 &= 34 + 47 \\ 47 + 34 &= 81 \\ 81 - 34 &= 47 \\ 81 - 47 &= 34 \\ 34 &= \square - 47 \end{aligned}$$

34 more than 47 is 81.
81 is 47 more than 34.
47 fewer than 81 is 34.
34 is 47 fewer than 81.

Addition (Augmentation)

Amber has 27 gel pens. She buys 30 more. How many gel pens does she now have?

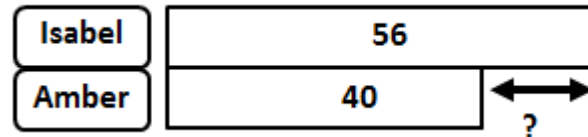
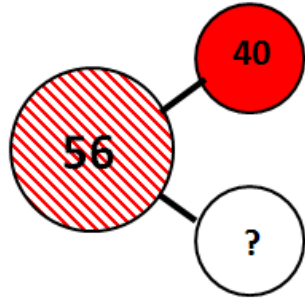


$$\begin{aligned} 57 &= 30 + 27 \\ 27 + 30 &= 57 \\ 57 - 30 &= 27 \\ 57 - 27 &= 30 \\ 30 &= \square - 27 \end{aligned}$$

30 more than 27 is 57.
57 is 27 more than 30.
27 more than 30 is 57.
30 fewer than 57 is 27.

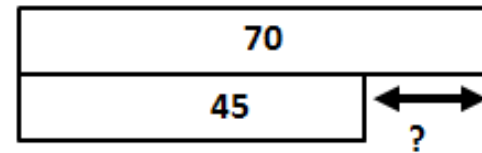
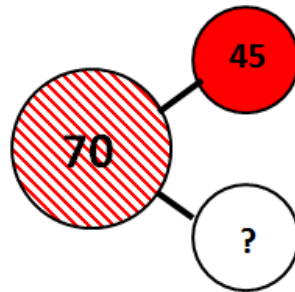
Subtraction (Comparison)

Amber has 40 buttons. Isabel has 56 buttons. How many fewer buttons does Amber have than Isabel?



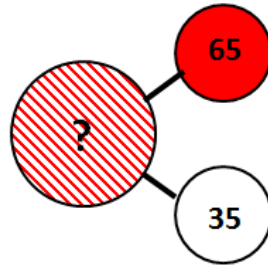
Further examples:

Amber needs 70g of flour for her cake. There is 45g of flour left in the bag. How much more flour does she need?



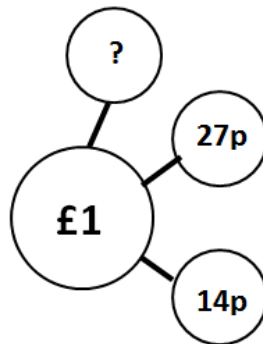
Further examples:

Tom has 65cm of ribbon. This is 35cm less than Sam has. How much ribbon does Sam have?



?	
65	35

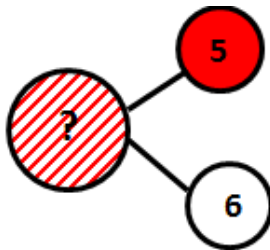
Ellie has £1 pocket money. She spent 27p on a pen and 14p on a rubber. How much money does she have left?



£1	
27p	14p

← ? →

I think of a number then subtract 6. The answer is 5. What was my number?



?	
5	6

- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems

Exploring commutativity and associative law.

$$5 + 2 + 1$$



5	2	1
---	---	---

$$1 + 5 + 2$$



1	5	2
---	---	---

$$1 + 2 + 5$$



1	2	5
---	---	---

Cuisenaire rods can be used to explore the generalised relationship of addition and subtraction.



Does red equal green subtract yellow?

Yellow is green add red.
Red add green equals yellow.
Yellow subtract green equals red.
Yellow subtract red equals green.

Sally writes an answer to the calculation below.

$$69 - 47 = 22$$

What calculations could she do to check her answer?

69	
22	47

$$\begin{aligned} 69 &= 22 + 47 \\ 47 + 22 &= 69 \\ 47 &= 69 - 22 \\ 69 - 47 &= 22 \\ 22 &= 69 - \square \end{aligned}$$

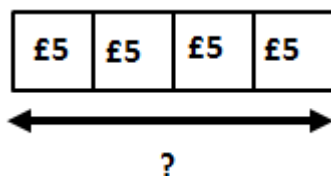
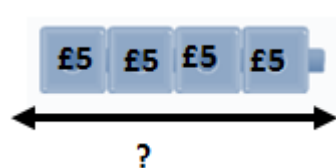
Multiplication and division

- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times) and division (\div) and equals ($=$) signs
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in contexts

Multiplication - Equal groups

(It is important that children start to understand unitisation at this point)

James buys four tickets for the football match. Each ticket costs £5. How much does he spend?

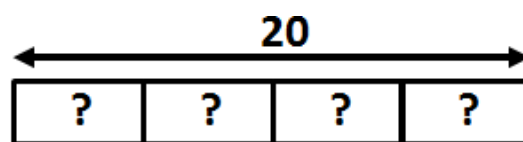
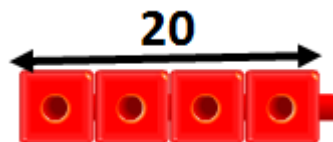


Four groups of £5 is £20.
£5 four times is £20.

$$\begin{aligned} \pounds 5 + \pounds 5 + \pounds 5 + \pounds 5 &= \pounds 20 \\ 4 \times \pounds 5 &= \pounds 20 \end{aligned}$$

Division – Sharing

There are 20 children in the hall. The teacher wants to put the children into 4 equal teams. How many children are in each team?

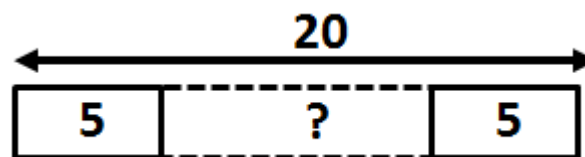
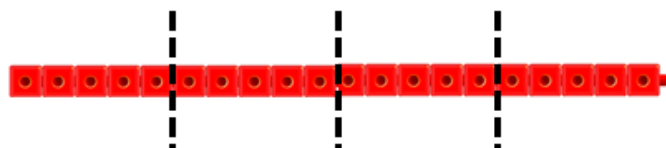


20 shared equally between 4 groups results in 5 in each group.

$$\begin{aligned} 20 \div 4 &= 5 \\ 4 \times 5 &= 20 \end{aligned}$$

Division: Grouping

Claire wants to put 5 biscuits on each plate. She has 20 biscuits. How many plates does she need?

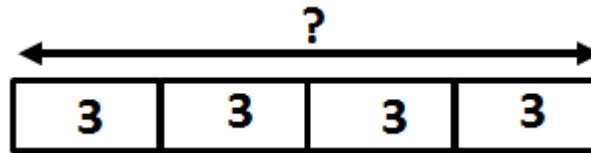
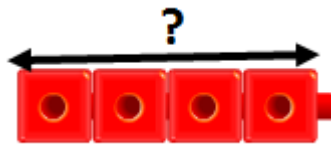


$$\begin{aligned} 20 \div 5 &= 4 \\ 4 \times 5 &= 20 \end{aligned}$$

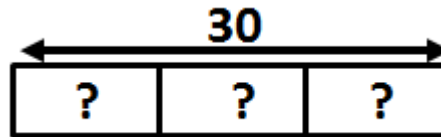
There are 4 groups of 5 in 20.

Additional examples

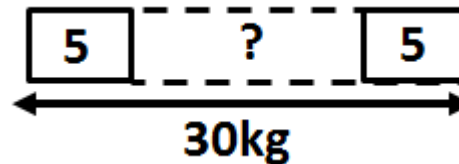
Claire has 4 building blocks. Each building block is 3m long. If she lays them end to end to build a wall, how long is the wall?



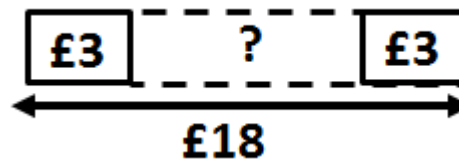
Amber has a ribbon 30cm long. She cuts it into 3 equal pieces. How long is each piece?



Isabel bought some bags of apples. Each bag has a mass of 5kg. The mass of all of the bags is 30 kg. How many bags of apples did Isabel buy?



Amber bought some books at the school fair for £18. Each book cost £3. How many books did she buy?



Fractions

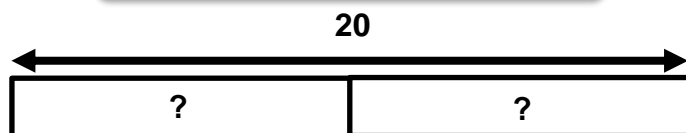
- Recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity

Explore the different fractions using Cuisenaire rods and multilinking cubes

Half



What is half of 20?



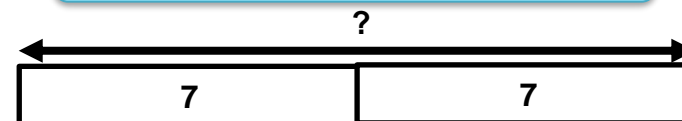
What fractions do you see?

If the orange is worth 6, what is the value of the yellow?

If each of the yellow rods is worth 8, what is the value of the orange?

If the orange is worth 100, what is the value of the yellow?

Half of a number is 7, what is the number?



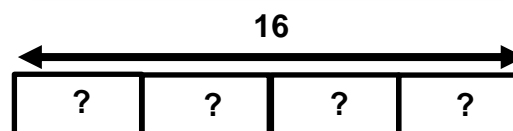
Repeat similar questions with the other fractions

Quarters

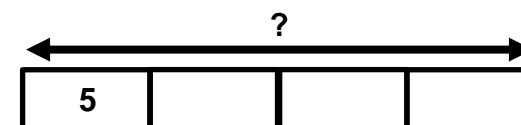


What is one quarter of 16?

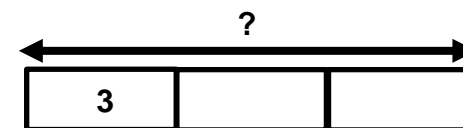
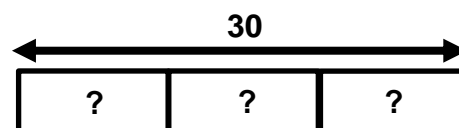
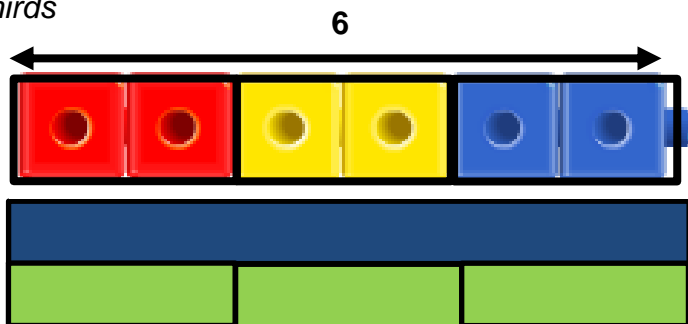
What is $\frac{3}{4}$ of 16?



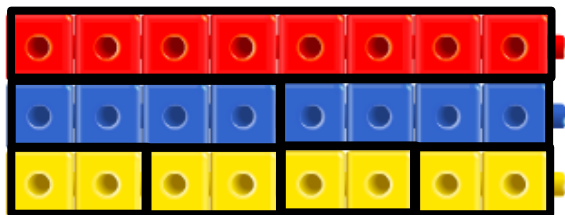
One quarter of a number is 5, what is the number?



Thirds



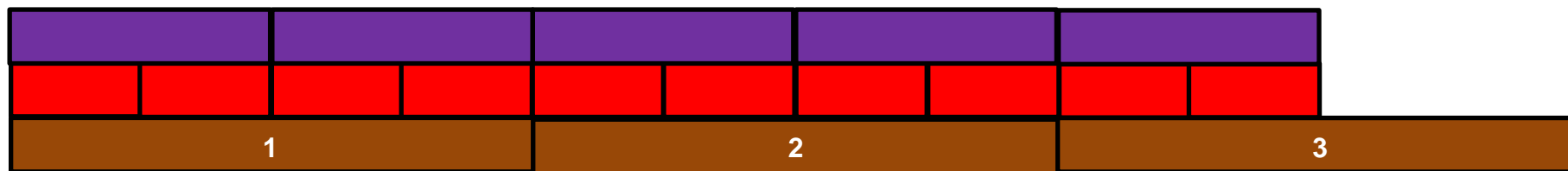
- Write simple fractions for example, $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$



Which other colour rods can you use to show both halves and quarters?

12			
6		6	
3	3	3	3

- Pupils should count in fractions up to 10 (*non-statutory*)



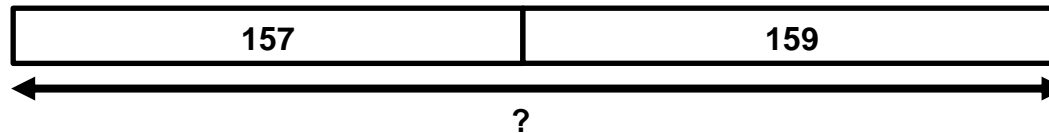
YEAR 3

Addition and Subtraction

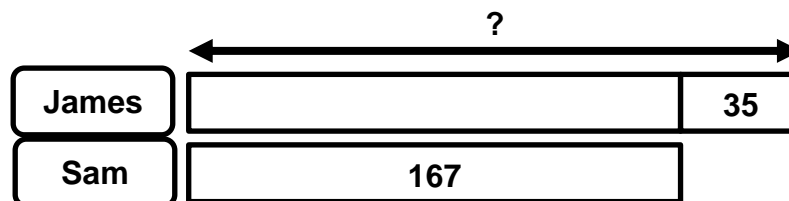
- Add and subtract numbers mentally, including:
 - a three-digit number and ones
 - a three digit number and tens
 - a three digit number and hundreds
- Estimate the answer to a calculation and use inverse operations to check answer
- Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction

Addition

In a cricket match, James' team score 157 runs in the first innings and 159 in the second innings. How many runs did they score in total?

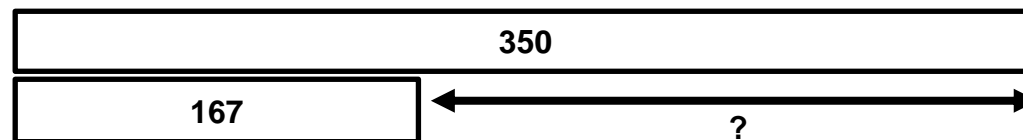


James scored 35 more points than Sam. Sam scored 167 points in his game. How many points did James score?

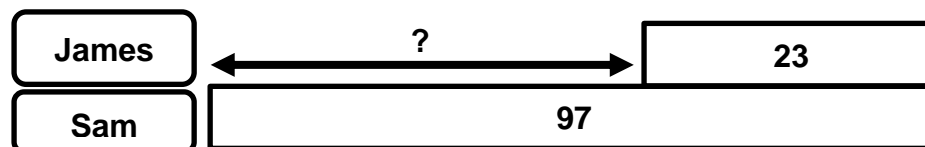


Subtraction

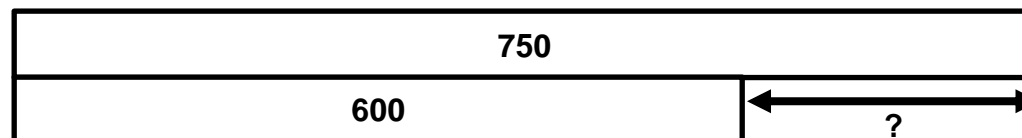
There are 350 pages in Amber's book. On Tuesday, she reads 167 pages of her book. On Wednesday, she reads the rest of the book. How many pages did she read on Wednesday?



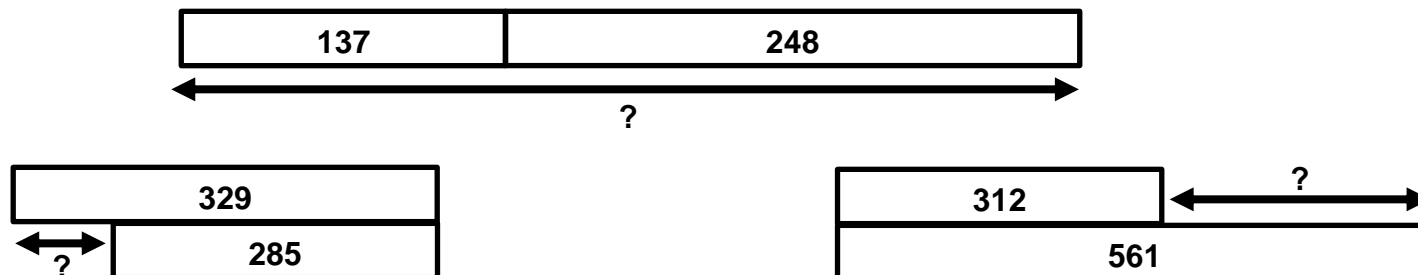
James has collected 23 fewer football stickers than Sam. Sam has collected 97. How many football stickers has James collected?



The bag of flour weighs $\frac{3}{4}$ kg. Nicola uses 600g of flour. How much flour



What story problems could these bar models represent?



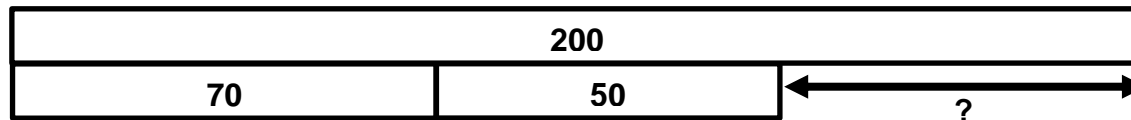
Check answers using the inverse

For the calculation $323 + 28$, Amber undertakes the calculation $351 - 323$.
Can you explain why she chose to do this?

351	
28	323

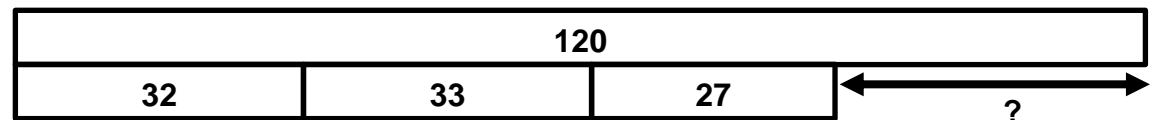
Multi-step problems

Gill has £2 pocket money. She spends 70p on a comic and 50p on a bar of chocolate. How much change should she get?



Complete this table:

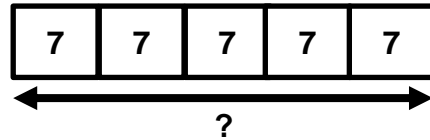
Year group	Number of children
3	32
4	33
5	?
6	27
Total	120



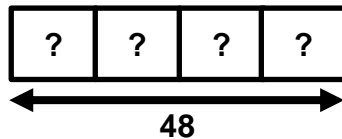
Multiplication and Division

- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

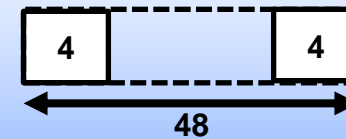
$$5 \times 7 = \square$$



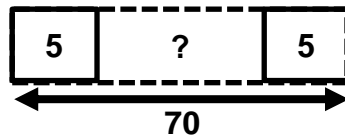
$$4 \times \square = 48$$



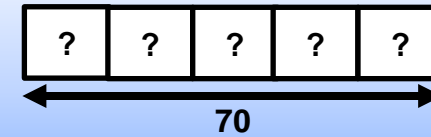
Or (due to commutativity)



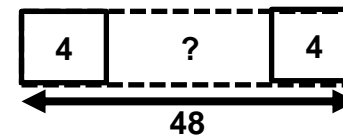
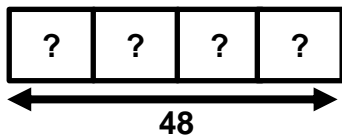
$$\square \div 5 = 70$$



Or (due to commutativity)



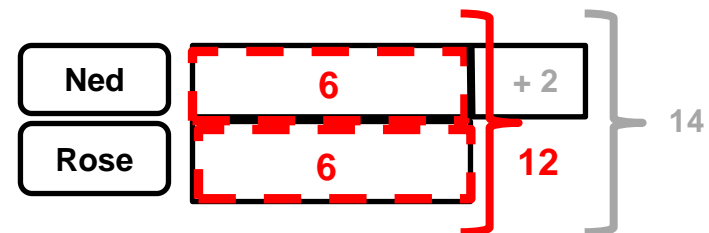
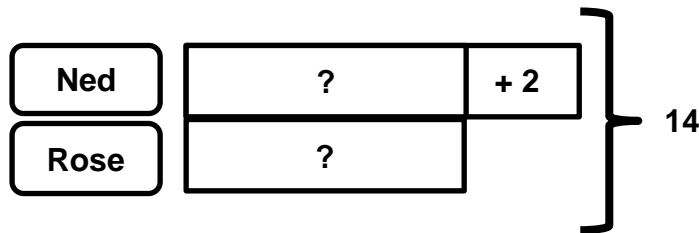
$$48 \div \square = 4$$



It could either be number of groups or group size that is unknown.

- Solve problems, including missing number problems, involving multiplication and division

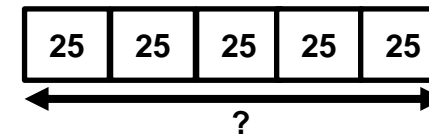
Ned and Rose have some biscuits. Altogether, they have 14 biscuits. Ned has 2 more biscuits than Rose. How many biscuits do they each have?



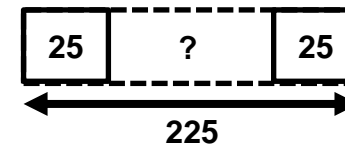
Measures

One length of the swimming pool is 25 metres.

a) Rachel swims five lengths of the pool. How far does Rachel swim altogether?

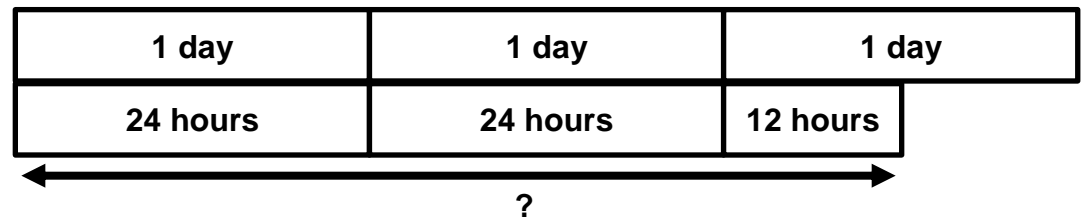


b) Sam swims 225m in the pool. How many lengths does he swim?



Time

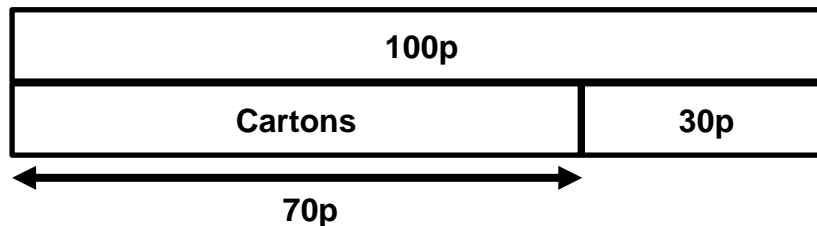
How many hours are there in two and a half days?



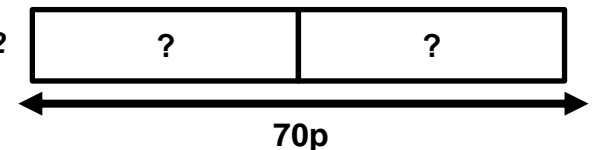
Money

I had one pound. I bought two cartons of drink and received 30p change. How much did each carton cost?

Step 1



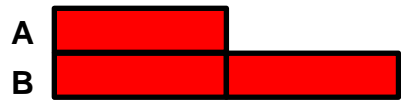
Step 2



Multiplication and Division

- including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

At this stage, focus upon the multiplicative relationship between the bars.



Look at the bar model.
What can you tell me?

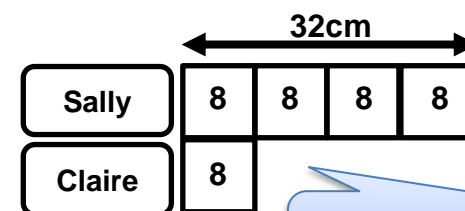
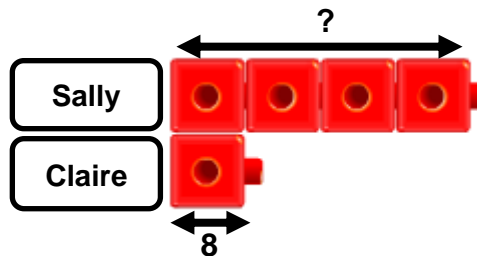
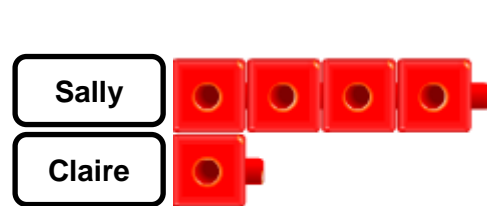


Can you change this model to show
that C is four times as big as D?

Provide a speaking frame if
required, for example:

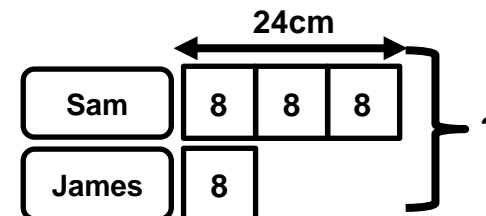
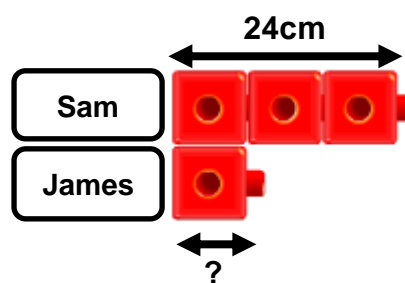
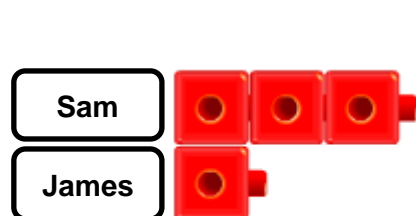
.... is twice as many as ...
... is three times as many as ...

Claire builds a tower that is 8 cubes tall. Sally builds a tower that is 4 times as tall. How tall is Sally's tower? How much smaller is Claire's tower?



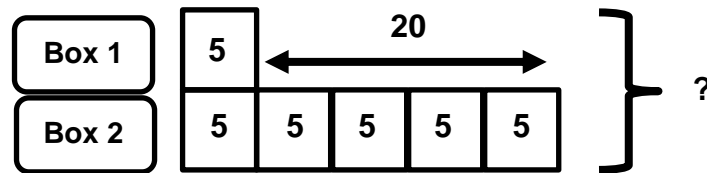
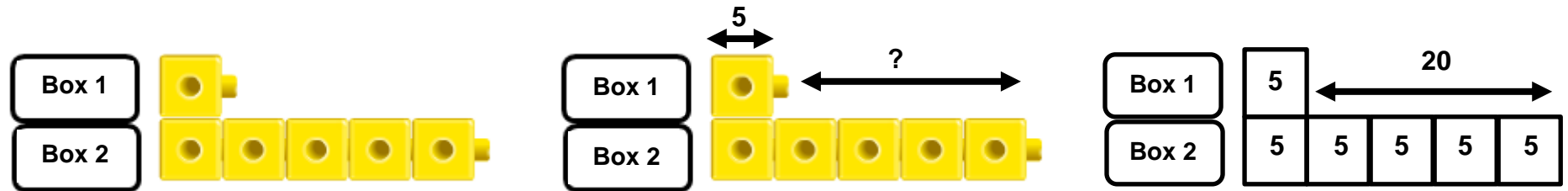
How much smaller is Claire's
tower than Sally's tower?

Sam's ribbon is three times as long as James's ribbon. Sam's ribbon is 24cm long. How long is James's ribbon?

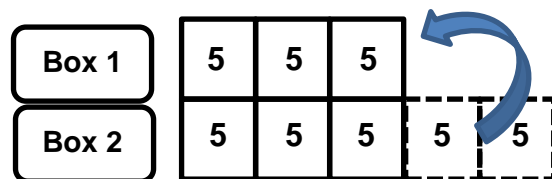


What is the total length
of their ribbons?

Leigh is helping in the school library. She is packing books into two boxes. The first box has 5 books in it. The second box has five times as many books in it as the first box has. How many more books does the second box hold than the first?



How many books, in total, have been packed into the two boxes?

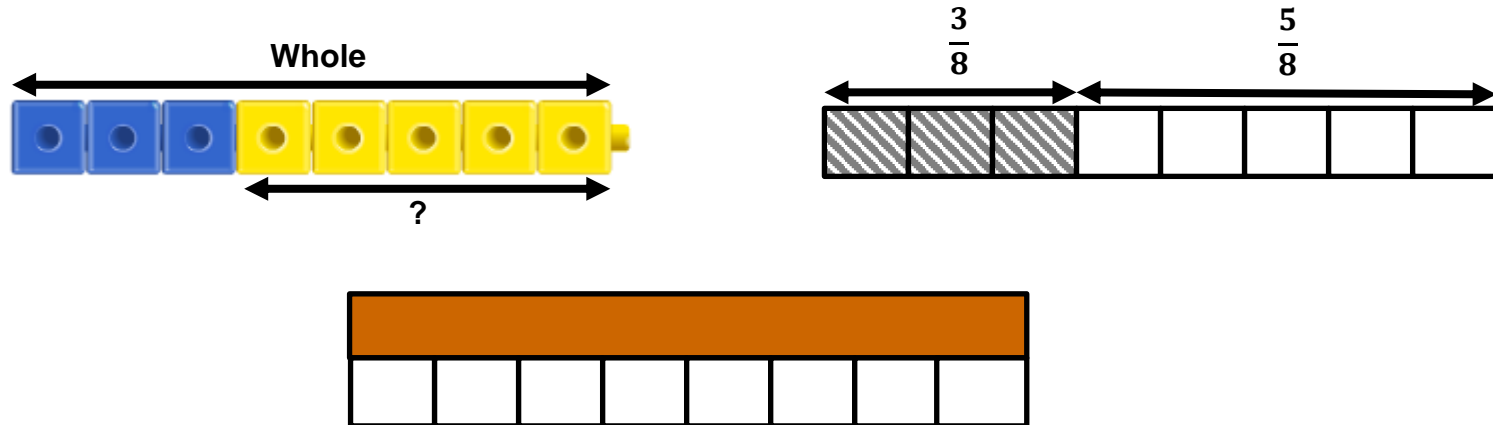


How many books need to be taken out of Box 2 and placed in Box 1 so that there is an equal number of books in each box?

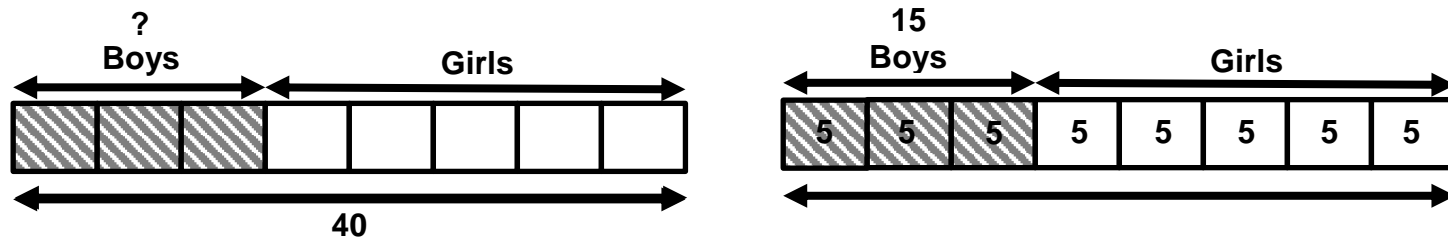
Fractions

- Recognise, find and write fractions of discrete set of objects; unit fractions and non-unit fractions with small denominators

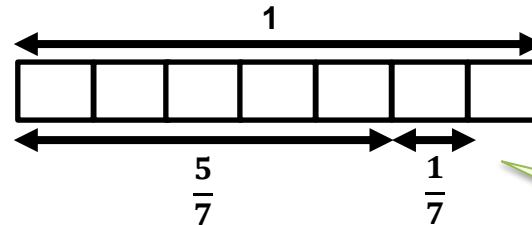
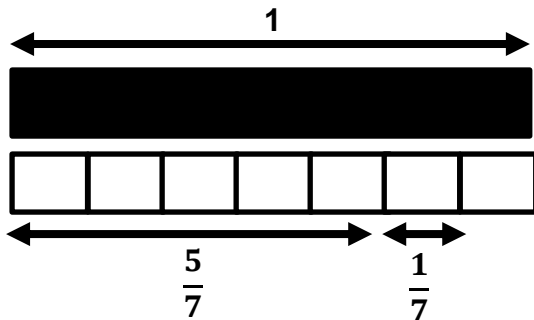
$\frac{3}{8}$ of a class are boys. What fraction of the class are girls?



If there are 40 children in the class, how many boys are there?



- Add and subtract fractions within the same denominator within one whole



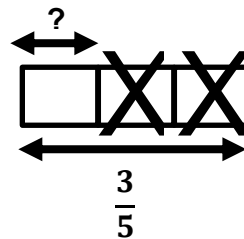
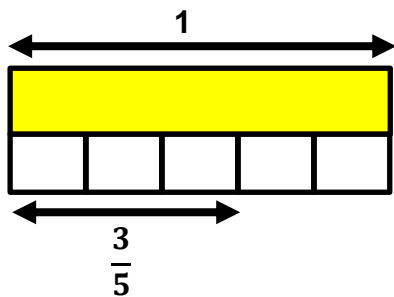
Ensure that the children define the fractions with reference to the whole.

$\frac{5}{7}$ add $\frac{1}{7}$ is equal to $\frac{6}{7}$ of the whole

Subtraction

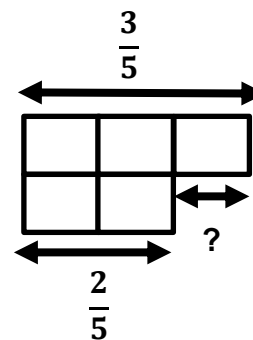
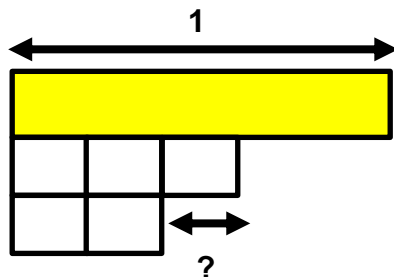
Take away model

$$\frac{3}{5} - \frac{2}{5} = ?$$



$\frac{3}{5}$ take away $\frac{2}{5}$ is equal to $\frac{1}{5}$ of the whole

Comparison model

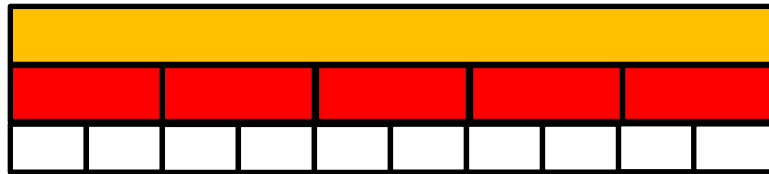


The difference between $\frac{3}{5}$ and $\frac{2}{5}$ is $\frac{1}{5}$ of the whole

- Recognise and show, using diagrams, equivalent fractions with small denominators

Children could explore this with Cuisenaire rods first and then with bar models

Find equivalent fractions to $\frac{2}{5}$



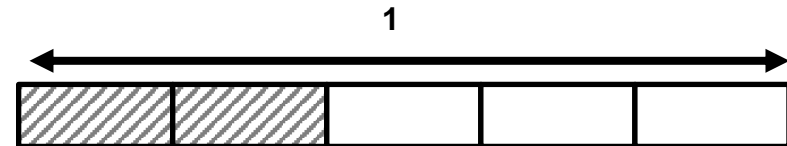
Ensure that the children define the fractions with reference to the whole.

What will it look like if we split the thirds into three equal pieces? What can we see now?

Find equivalent fractions to $\frac{2}{3}$



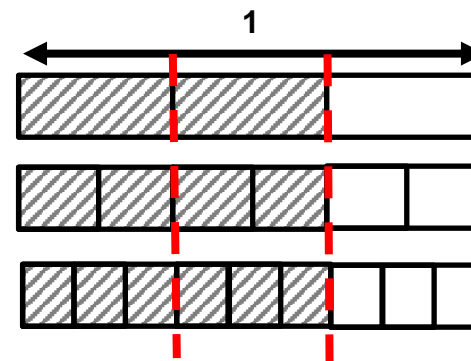
What does $\frac{2}{5}$ look like as a bar model?



Let's split each of the fifths into two equal pieces. What does that look like?



What do you see now?

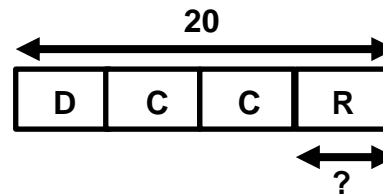


$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$$

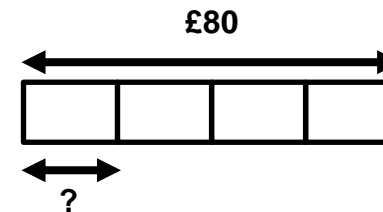
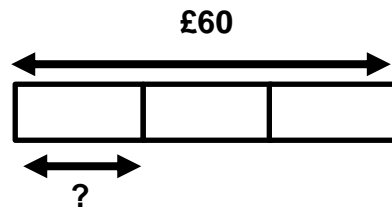
This conversion can be more easily explored pictorially.

- Solve problems that involve all of the above

Sally has 20 stickers on her page. One quarter of them are dog stickers. One half of them are cat stickers. The rest are rabbit stickers. How many rabbit stickers are on the page?



What is the larger amount, one third of £60 or one quarter of £80?

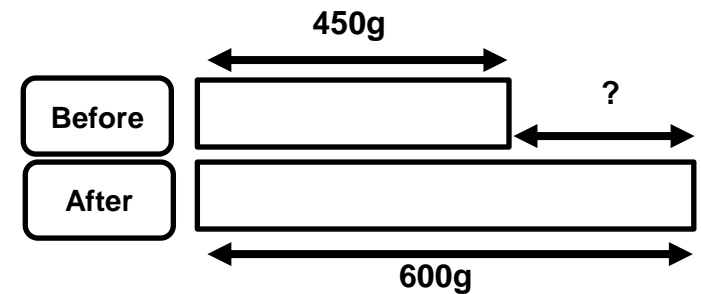
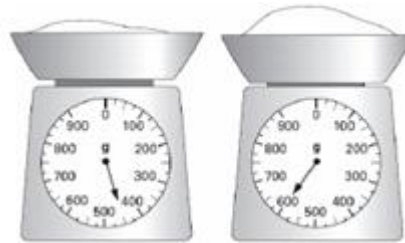


YEAR 4

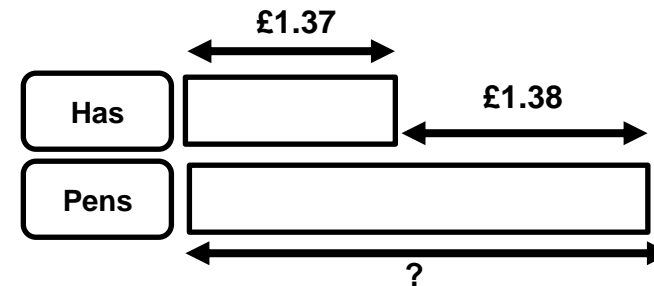
Addition and Subtraction

- Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

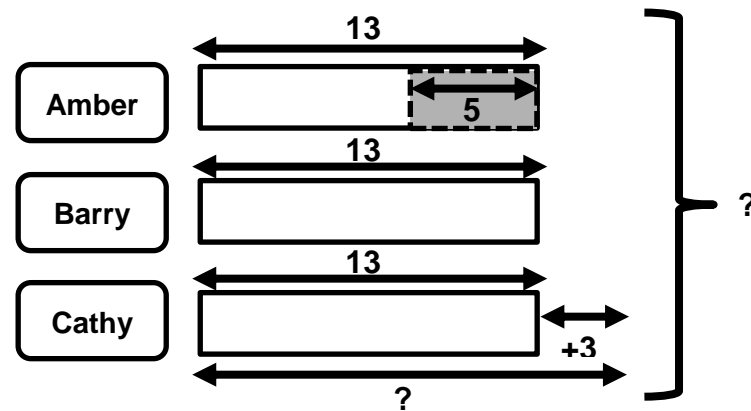
Emily is making a cake. She puts flour on the scales. How much sugar does she add?



Claire has £1.37. She needs £1.38 more to buy a packet of pens. How much do the pens cost?



Amber, Barry and Cathy run a 50m race. Barry's time is 13 seconds. Amber finishes 5 seconds before Barry. Cathy finishes 3 seconds after Barry. What is Cathy's time?

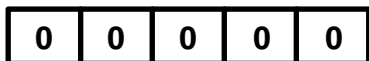


What is the total time taken by all three children?

Multiplication and Division

- Including multiplying by 0 and 1; dividing by 1; multiplying together three numbers

$$5 \times 0$$



What does this mean?

Use a speaking frame to help support pupils to develop the range of appropriate language.

Number of groups x size of group = total number of objects.

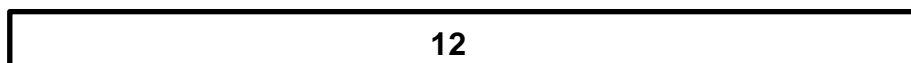
Explore concrete ideas such as 5 plates with no biscuits.

How many biscuits?

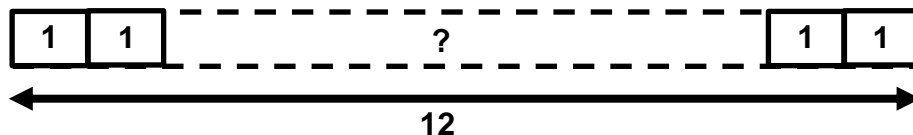
Division

$$12 \div 1$$

Sharing

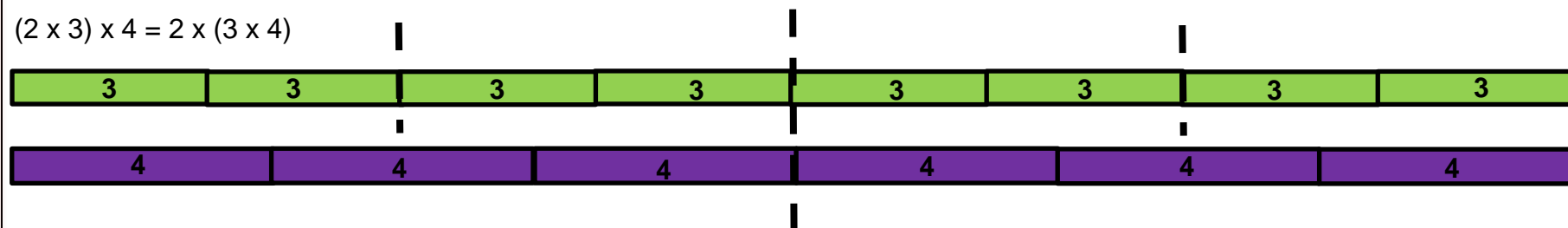


Grouping



Multiply three numbers together – **Associative law**

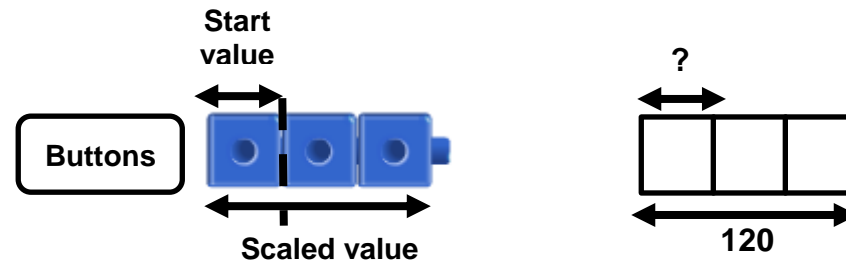
$$(2 \times 3) \times 4 = 2 \times (3 \times 4)$$



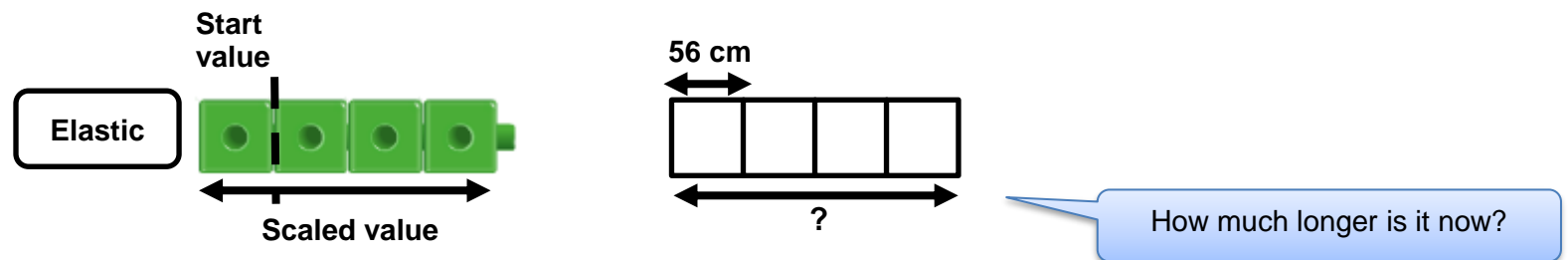
- Solve problems involving multiplication and adding, including using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Integer scaling

Louise collects buttons. She bought some new buttons to add to her collection. She now has 120 buttons, which is three times as many buttons as she started with. How many buttons did she start with?

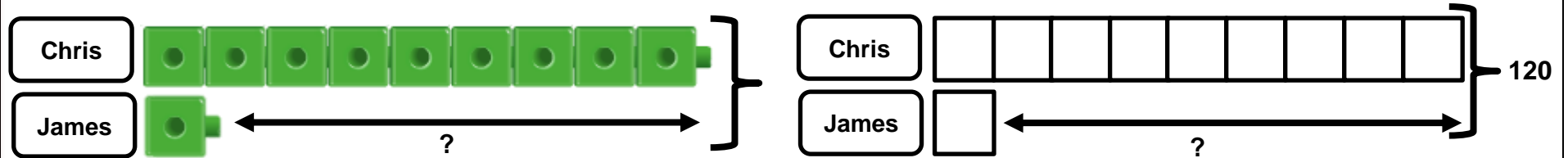


A length of elastic is stretched four times longer than its original size. It was 56cm long. How long is it now?



Correspondence problems

Chris has 9 times as many Lego figures as James. Together they have 120 Lego figures. How many more Lego figures does Chris have than James?



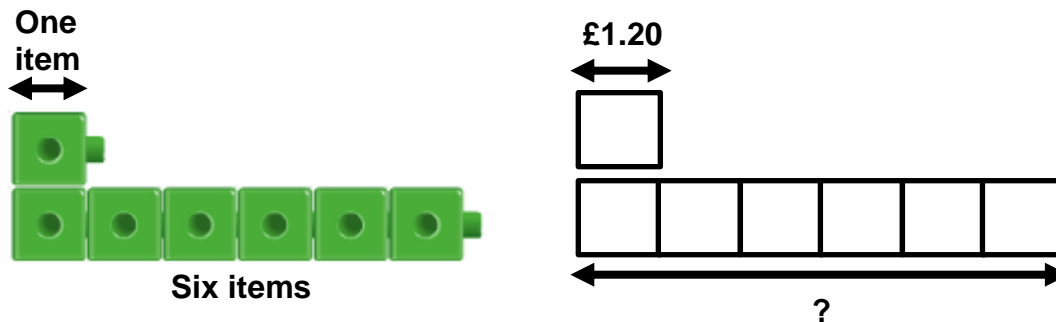
How many Lego figures would Chris have to give James so that they have an equal amount?

Claire has twice as many stickers as her friend Emma. Claire has 90 stickers. How many stickers do they have together?



Rate – this is not a statutory unit Year 5

An ice cream costs £1.20. Louise wants to buy 6 ice creams. How much will the ice creams cost?



If Louise pays with a £10 note, how much change will she receive?
If Louise only has £1.50, how much more money does she need?

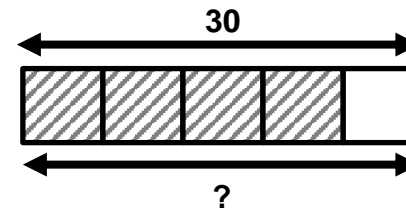
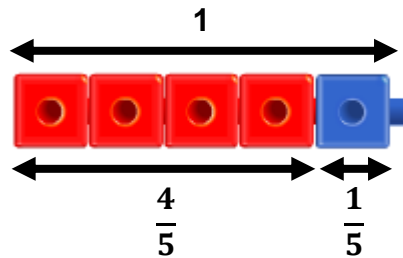
Fractions

- Recognise and show, using diagrams, families of common equivalent fractions

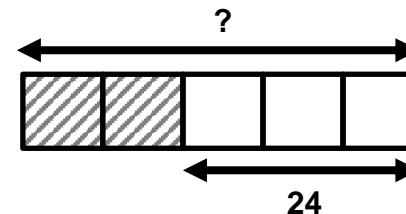
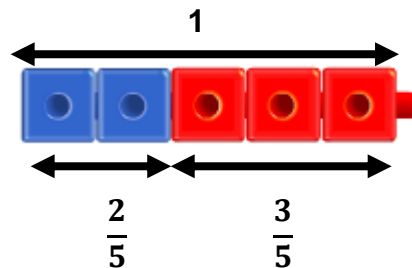
See Year 3 progression

- Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number

Sally buys four fifths of the shop's apples. If the shop had 30 apples, how many apples did she buy?



James had some football cards. He gave two fifths away. He now has 24 cards. How many did he have to start with?



- Add and subtract fractions with the same denominator

See year 3 progression

YEAR 5

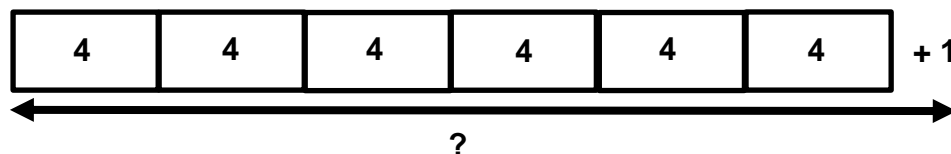
Multiplication and Division

- Divide numbers ... interpret remainders appropriately for the context

The aim of these examples is to support **understanding the bar model with remainders**

Multiplication

Tracey put 4 seeds into each of her pots. She uses 6 pots and has 1 seed left over. How many seeds did she start with?



Why is the 'one' not in a block?

Grouping

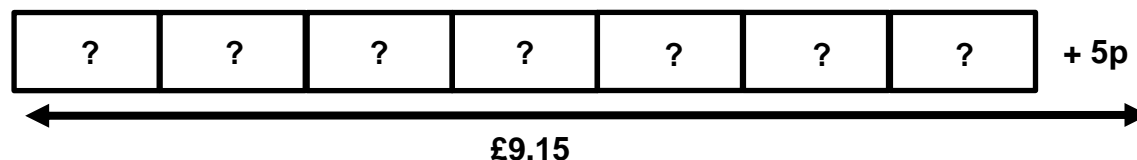
Carl has 580 sweets. He wants to put 70 sweets in each bag. How many bags can he fill?



What does this number mean?
How many sweets would Carl have left for himself?
Carl has 20 sweets left.

Sharing

Grandad has been collecting his change in a jar. He has £9.15. He wants to share the money equally between seven grandchildren. How much money will each child receive?

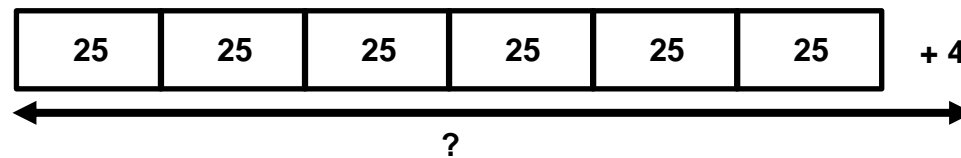


Why does the line have to include the remainder?

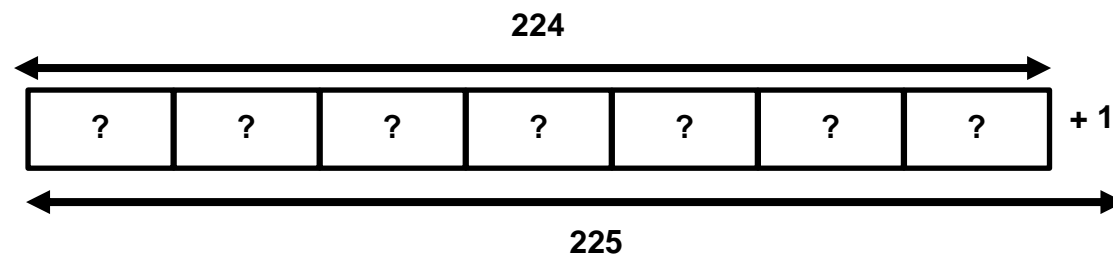
Exploring deep understanding of the division notation

- Solve:

$$\square \div 6 = 25 \text{ r } 4$$



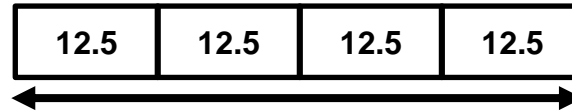
$$225 \div \square = 7 \text{ r } 1$$



- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the equals sign

Every day, for 4 days, Sally scored 12.5 on her test. On her fifth day, she scored 14. What was her total score for the week?

○ Step 1

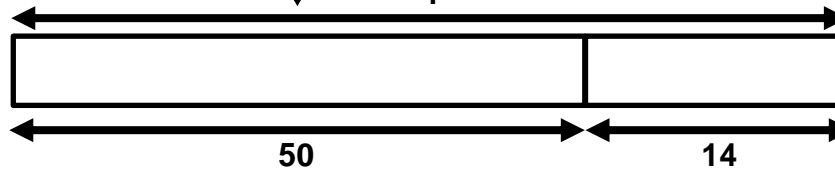


50



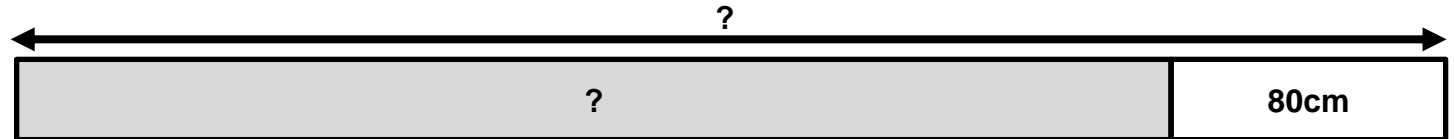
?

○ Step 2



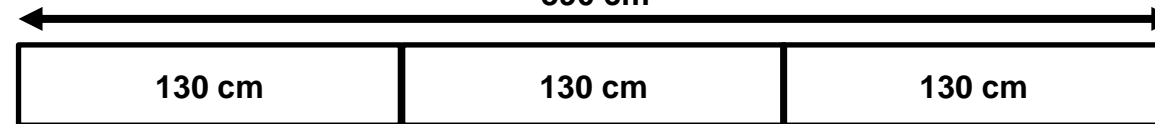
I cut 80cm from a length of ribbon and shared the remainder between 3 friends. Each friend now has 1.3m of ribbon. How much did I start with?

○ Step 1

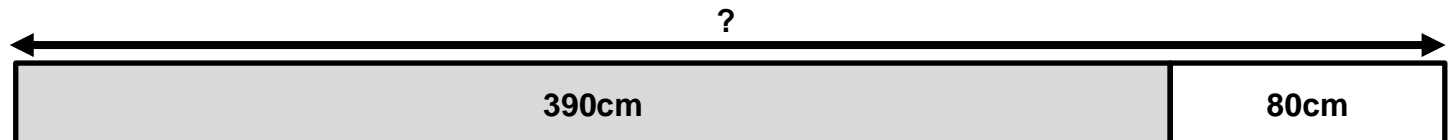


390 cm

○ Step 2

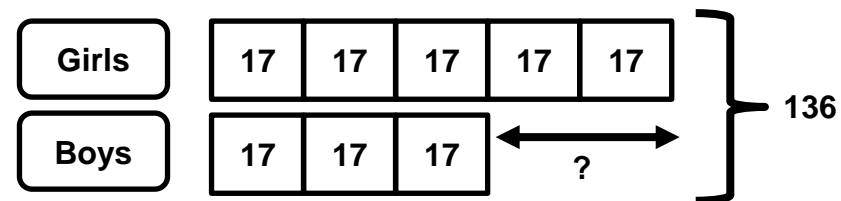


○ Step 3



Complete the original model with the information from step 2.

At the school disco, there are 5 girls to every 3 boys. If there are 136 children at the disco. How many more girls than boys are there?



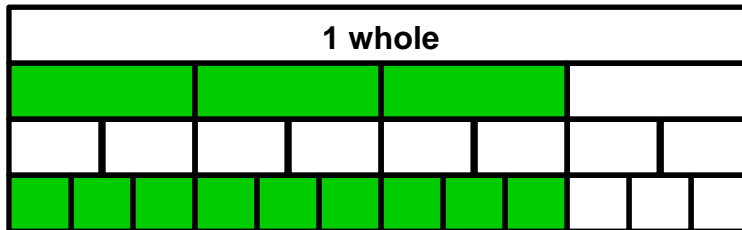
Fractions

- Compare and order fractions whose denominators are all multiples of the same number

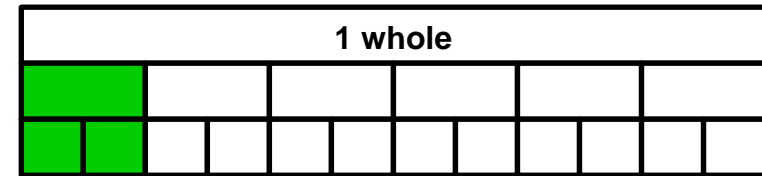
The aim is for the children to discover the relationship and the rule for themselves (see Year 4).

Which fraction is greater $\frac{3}{4}$ or $\frac{1}{6}$?

Three quarters converted into twelfths



One sixth converted into twelfths



You can compare fractions that have the same numerator by comparing their denominators.

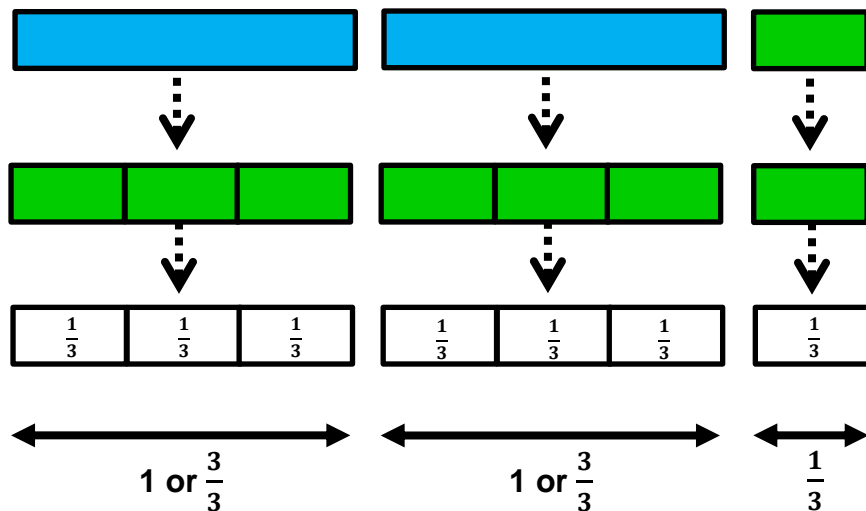
- Which is bigger $\frac{3}{5}$ or $\frac{3}{6}$?

You can compare fractions that have the same denominator by comparing their numerators.

- Which is smaller $\frac{2}{10}$ or $\frac{4}{10}$? How much smaller?

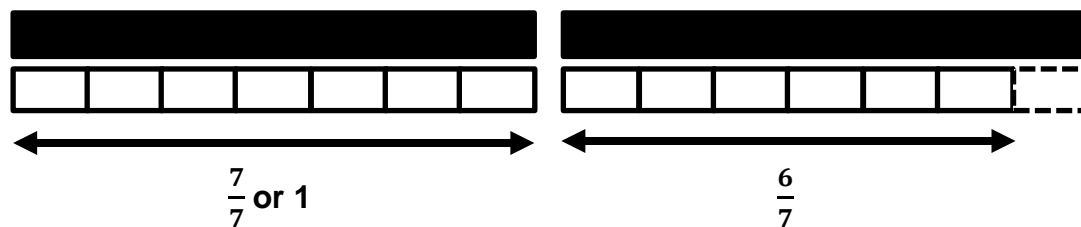
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements >1 as a mixed number

Convert $2\frac{1}{3}$ into an improper fraction (Cuisenaire rods experience)



$$= \frac{7}{3}$$

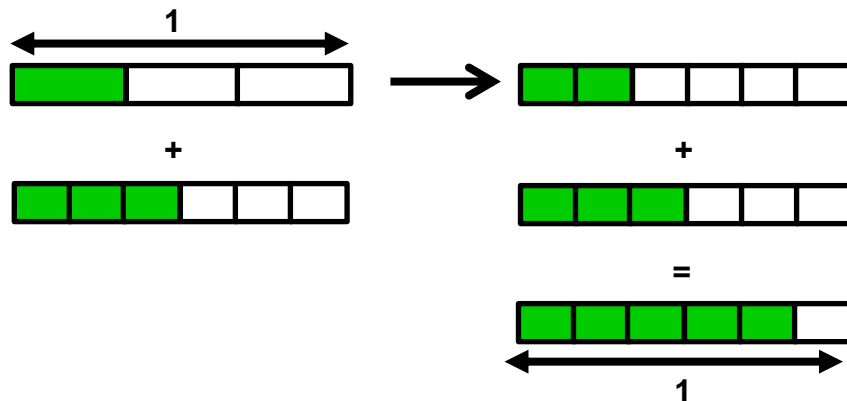
Convert $\frac{13}{7}$ to a mixed number



$$\frac{13}{7} = 1\frac{6}{7}$$

- Add and subtract fractions with the same denominator and multiples of the same number

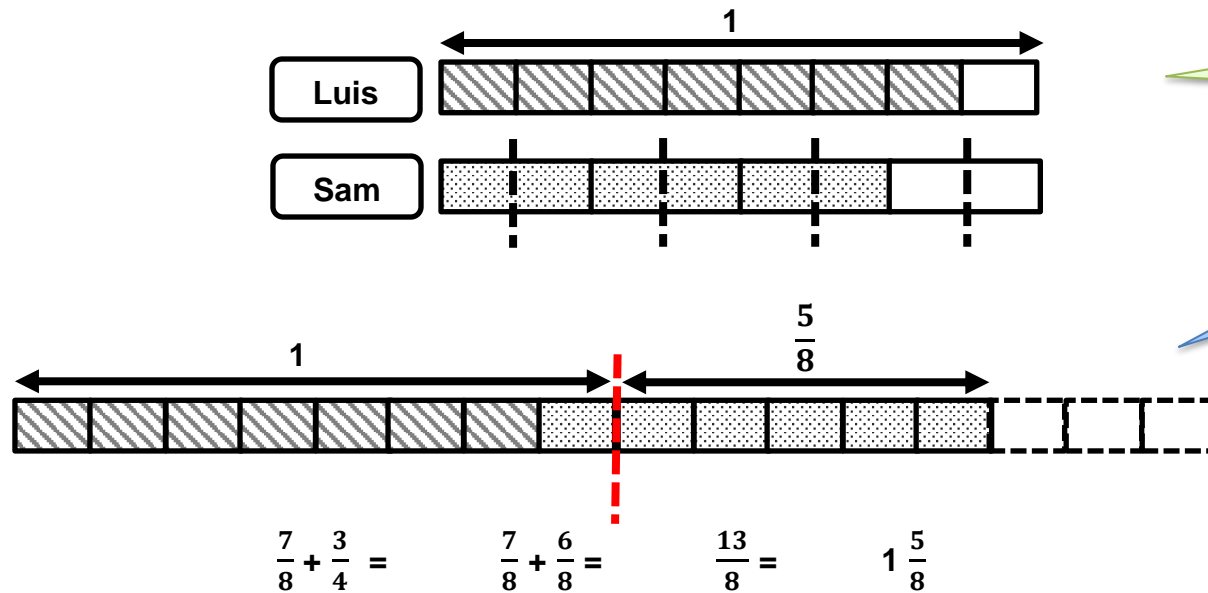
$$\frac{1}{3} + \frac{3}{6}$$



$\frac{1}{3}$ is equivalent to $\frac{2}{6}$

$\frac{3}{6}$ added to $\frac{2}{6}$ equals $\frac{5}{6}$

Sam and Luis have a pizza each. Sam eats $\frac{3}{4}$ of his pizza. Luis eats $\frac{7}{8}$ of his pizza. How much pizza have they eaten altogether?

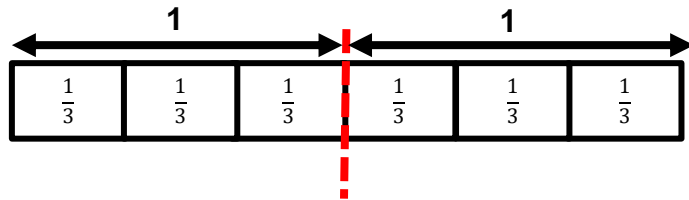


$\frac{3}{4}$ is equivalent to $\frac{6}{8}$

How much pizza is left?

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

$$6 \times \frac{1}{3} = ?$$

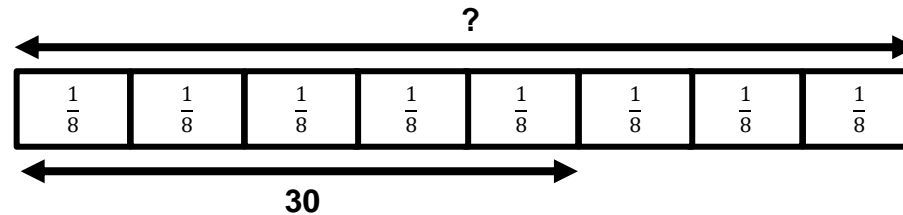


$$= \frac{6}{3} = 2$$

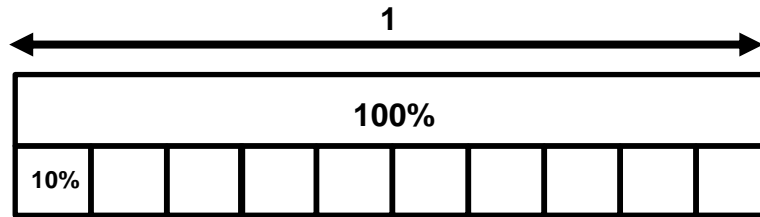
How could you say this?
Refer the children back to the speaking frame.
Number of groups x size of each group = product.

- Solving problems involving fractions

30 is $\frac{5}{8}$ of a number. What is the number?



- Recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator hundred, and as a decimal

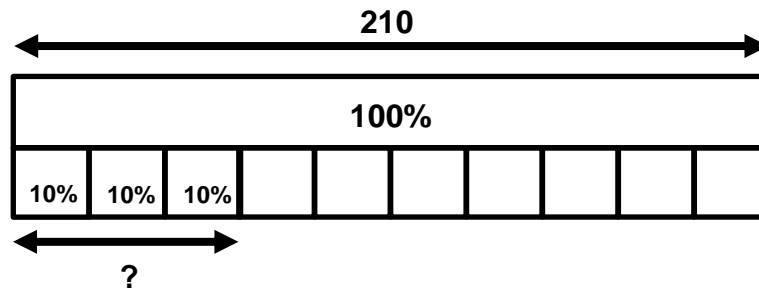


If the whole is 100 %, what does each block represent?
What if wanted each block to represent 20%, what would the model look like?

$$10\% = \frac{10}{100} = \frac{1}{10} = 0.1$$

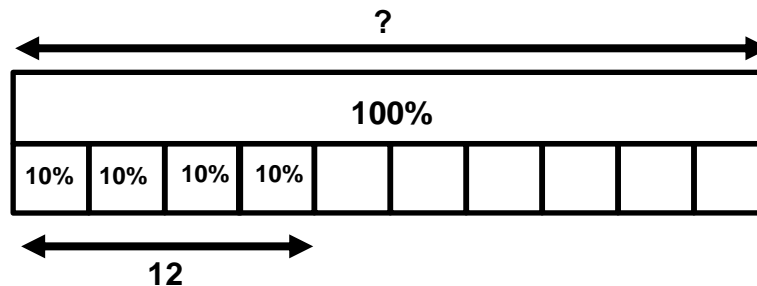
- Solve problems which require knowledge of percentage

- What is 30% of 210?



If the whole is 100 %, what does each block represent?
What is the value of each block?

- Twelve is 40% of a number. What is the number?

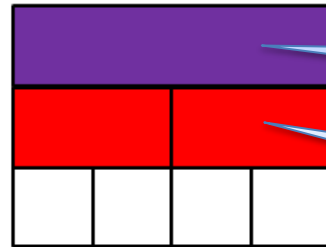


YEAR 6

- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

$$\frac{1}{2} + \frac{1}{4} = ?$$

In order to be able to add these fractions, the children need to convert them into equivalent fractions. Using Cuisenaire rods can support them in refreshing their knowledge of equivalent fractions.

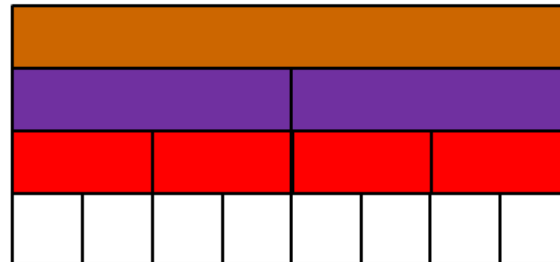


$$\frac{1}{2} + \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

The purple rod has been selected as this will allow the children to see one half and one quarter on the same model

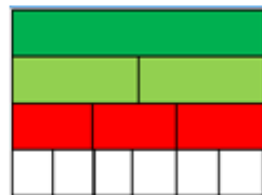
If purple is the whole, what colour shows us half? (red)
What colour rod would show us quarters?
Can we rename half? Yes, two quarters.

Deepen understanding by asking the children if there are other colour rods which would allow us to show halves and quarters on the same model.



$$\frac{1}{2} + \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

$$\frac{1}{2} + \frac{1}{3} = ?$$



$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Which colour rods will allow us to represent halves and thirds on the same model?
Can you explain why?
If the children need to allow them to explore.

- Multiply simple pairs of proper fractions, writing the answer in its simplest form

▪ $1 \times \frac{1}{4} =$



Why have I selected the purple rod?
What value does it represent?

How can I show $\frac{1}{4}$ of the purple rod?



▪ $3 \times \frac{1}{4} =$

If purple represents one, how many do I need?

So what is the answer?

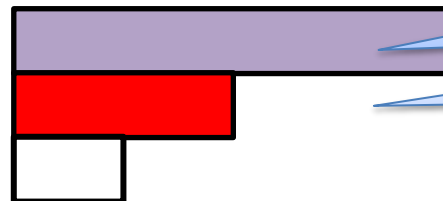


▪ $\frac{1}{2} \times \frac{1}{2} =$

If purple equals the whole...

What is half?

So white is equivalent to one half of the group of a half.
But what is its value?



In order to find the value of white, the children need to refer back to the whole.



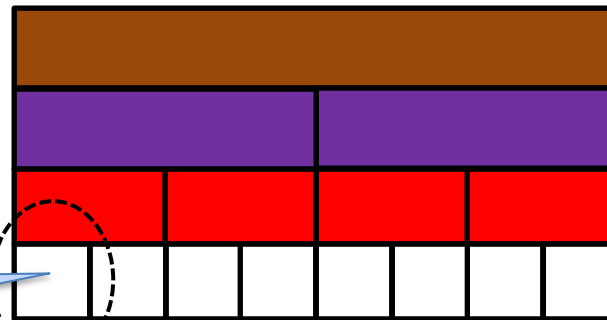
White is one quarter

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

▪ $\frac{1}{4} \times \frac{1}{2} =$

In this case, we need to discover how big one quarter group of one half is.

$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$



Where is one quarter of one half?

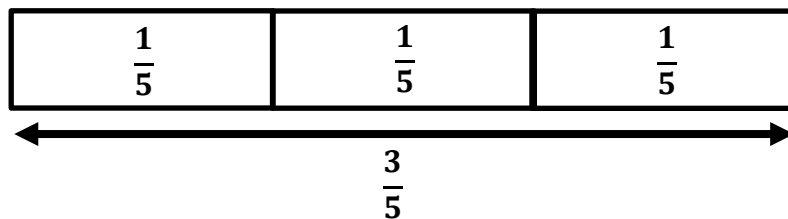
What is its value?
 $\frac{1}{8}$

What is one half of one quarter?
(application of commutativity)

- Divide proper fractions by whole numbers

▪ $\frac{3}{5} \div 3 =$

If you share _____ equally between ____ groups, how many are there in each group?



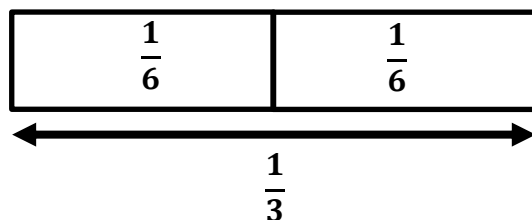
What else can you see?

I can see that $\frac{1}{5}$ is $\frac{1}{3}$ of $\frac{3}{5}$.

I can see that $3 \times \frac{1}{5}$ is equal to $\frac{3}{5}$.

I can see how many fifths are in three fifths.

▪ $\frac{1}{3} \div 2 =$

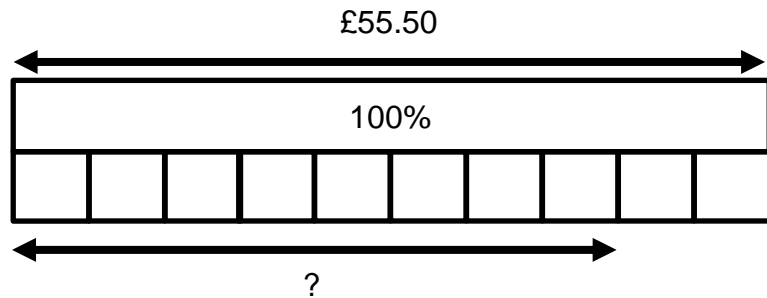


What else can you see?

I can see that half of one third is one sixth.
I can see double one sixth is equal to one third.
I can see two sixths are equivalent to one third.
I can see that there are two sixths in a third.

- Solve problems involving the calculation of percentages

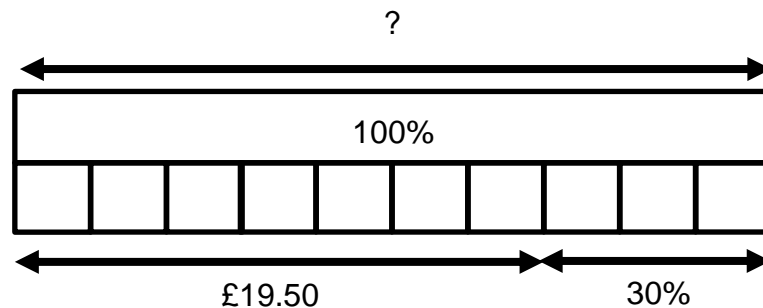
The normal price for a tracksuit is £55.50. How much will it cost in a '20% off' sale?



What is the value of 10%?
How can this be represented on the model?

Can you think of two ways to find the answer?

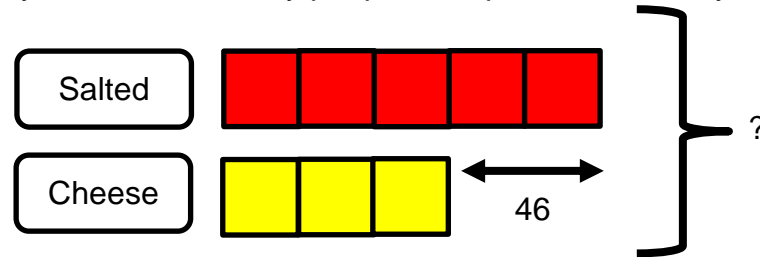
In a '30% off' sale, Carl pays £19.50 for a T-shirt. How much was the T-shirt before the sale?



How much has he saved?

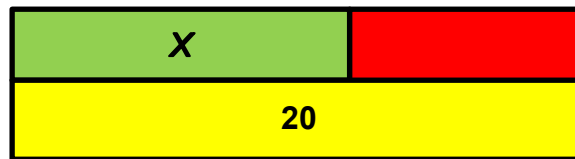
- Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

In a survey, the ratio of the number of people who preferred 'ready-salted' to 'cheese and onion' crisps was 5:3. Forty-six more people preferred ready-salted. How many people took part in the survey?

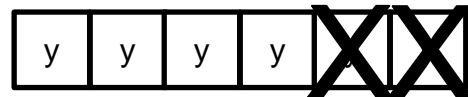


- Express missing number problems algebraically

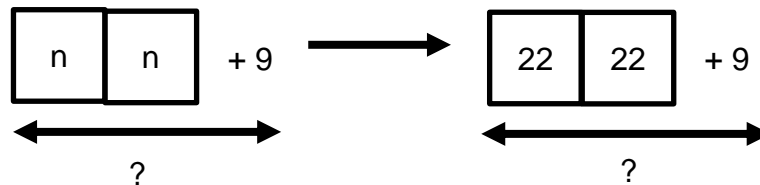
$$X + 5 = 20$$



$$6y - 2y = 4y$$



What is $2n + 9$ when $n = 22$





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