



Progression in Mental Mathematics

Primary Maths

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HfL Progression in Mental Mathematics Guidance

Rationale

This document was written in response to the heightened demands of the National Curriculum (2014). It aims to support teachers and leaders with a map of progression in mental fluency that is underpinned by research. The programme of study includes references to mental calculation but lacks the detail needed to provide a coherent pathway. It is up to schools to decide upon what this should look like. This guidance document provides the necessary detail.

With the expectation that pupils will move at 'broadly the same pace' through the curriculum, schools have been seeking a pathway from EYFS to Year 6 that will support the development of all pupils' mental skills, including those who might previously have struggled to develop the core knowledge and understanding necessary. This document reflects the findings of our research projects and 'tried and tested' approaches with schools, focusing on practice that is most effective at increasing mental fluency for more of our pupils.

Approaches

Working in collaboration with teachers, Herts for Learning advisers have found that the following approaches have had significant impact upon pupils' ability to develop number sense and multi-strategy approaches to mental calculation.

- **A risk-free environment where learning is valued over performance**

Where the environment praises speed and 'first to get the answer right', it emphasises a competitive view of mathematics. Unfortunately, this has the effect of 'hiding' how fluency is developing in other pupils and implies that mental calculation is a performance. This can adversely affect pupils' desire to engage. Instead, we promote a range of approaches that are more effective in engaging pupils to discuss and reason about their strategies. A risk-free classroom has an ethos that is underpinned by the following attributes:

1. *Everyone* has something to contribute and we *all* value those contributions
2. An appreciation that we each see things differently – there may be one answer, but there are a myriad of available journeys
3. This is not about guessing what is in the teacher's head
4. There is an expectation that we have to try to communicate our ideas so that everyone else can understand them and that we are expected to try and understand the thinking of others
5. There is an expectation that we have to listen to what others say and then try to build on it - agreeing and disagreeing by offering proof.

All of the following approaches can be utilised in shared, whole-class learning discussions. It is not an exhaustive list but provides a flavour of available possibilities.


1. Give the calculation *and* the answer – shared class discussion: 'How could you do this?'
2. 'One finger, one way' – show me your thumb when you have found one way to find the answer, keep thinking and show me another finger when you have found another way...
3. 'Show me, show me' – show me your thinking in as many ways as possible.
4. 'Can you use the _____ (*named*) strategy to solve this?'
5. 'Cluster of facts' – pupils identify facts that would be helpful to solve a calculation and / or explain why given facts might be useful to solve the problem.
6. 'Shortcuts' – "I could take a shortcut in this strategy if I ..."
7. 'Seek and destroy' – identify correct and incorrect answers from a range and explain why.

- **CPA (concrete-pictorial-abstract)**


To understand the numbers they are working securely with and develop number concepts alongside the procedures, the CPA approach allows pupils to demonstrate and explore learning across a range of representations.

For example, when very young pupils learn about '3 + 2' they need to learn that the symbols stand for the operation of addition i.e. adding 2 to 3. They also need to understand the concept of a sum. In the CPA approach, pupils would explore the calculation using **concrete apparatus** to identify the '2', the '3' and the sum 5 *as well as pictorial representations* of the same calculation and the **abstract notation** (including language) to better understand both the procedure of adding 2 to 3 *and* the idea of sum.

$3 + 2 = 5$




"3 cubes and 2 cubes equal 5 cubes"




"3 cubes and 2 more cubes is 5 cubes"

"five is two more than three"



"three girls and two boys is five children altogether"



"3 girls and 2 more girls makes 5 girls"

"What other facts could we use to make 5?"

Without exploration through a range of representations, we cannot expect pupils to develop a full understanding of the underpinning concepts, facts and skills that are integral to developing good mental fluency. Schools should decide on core representations. They also need to ensure that variations of these are also included so that once pupils are able to they can be supported to assimilate learning to new representations.

- **Practice**

Practice is a key approach to developing the automaticity needed to reduce cognitive load. Pupils who have facts and skills at their fingertips are more likely to attend to the particulars of new learning than those that do not. These pupils have to work harder and are over-burdened. At Herts for Learning, we think of practice not as meaningless repetition of facts in which pupils chant without thought or as a series of isolated facts learnt at home then tested in school, but as a chance to rehearse them within exercises that develop better thinking. Practice is an opportunity to keep facts and skills ‘simmering’ and a further chance to vary the ways that they are presented. Schools should be mindful of the *quality* of practice rather than the *quantity*. Similarly, they are advised to focus upon the facts and skills that will make the greatest difference to mental fluency at each phase.

- **Facts to be practised**

At the end of each of phase, further guidance identifies which facts will support fluency. For example:

Year Three Recall

- Sums and differences between pairs of numbers which are multiples of 10 and 100
- Doubles and halves of multiples of 10 or 100
- Complements to 100
- Complements to 60 (time)
- Complements of tenths that make 1
- Complements of fractions with the same denominator that make 1 e.g. $\frac{3}{7} + \frac{4}{7} = 1$
- x 3, x 4, x 8 multiplication facts including division facts
- Number of seconds in a minute
- Number of days in a month and in a year including a leap year

- **Skills to be practised**

HfL advisers have identified a selection of key skills that, when practised, lead to increased mental fluency. Alongside increasing fact acquisition, they allow pupils to develop greater access to choices of strategy.

These are denoted in the progression document by a blue lozenge

Core skill: REGROUPING

and are defined below.

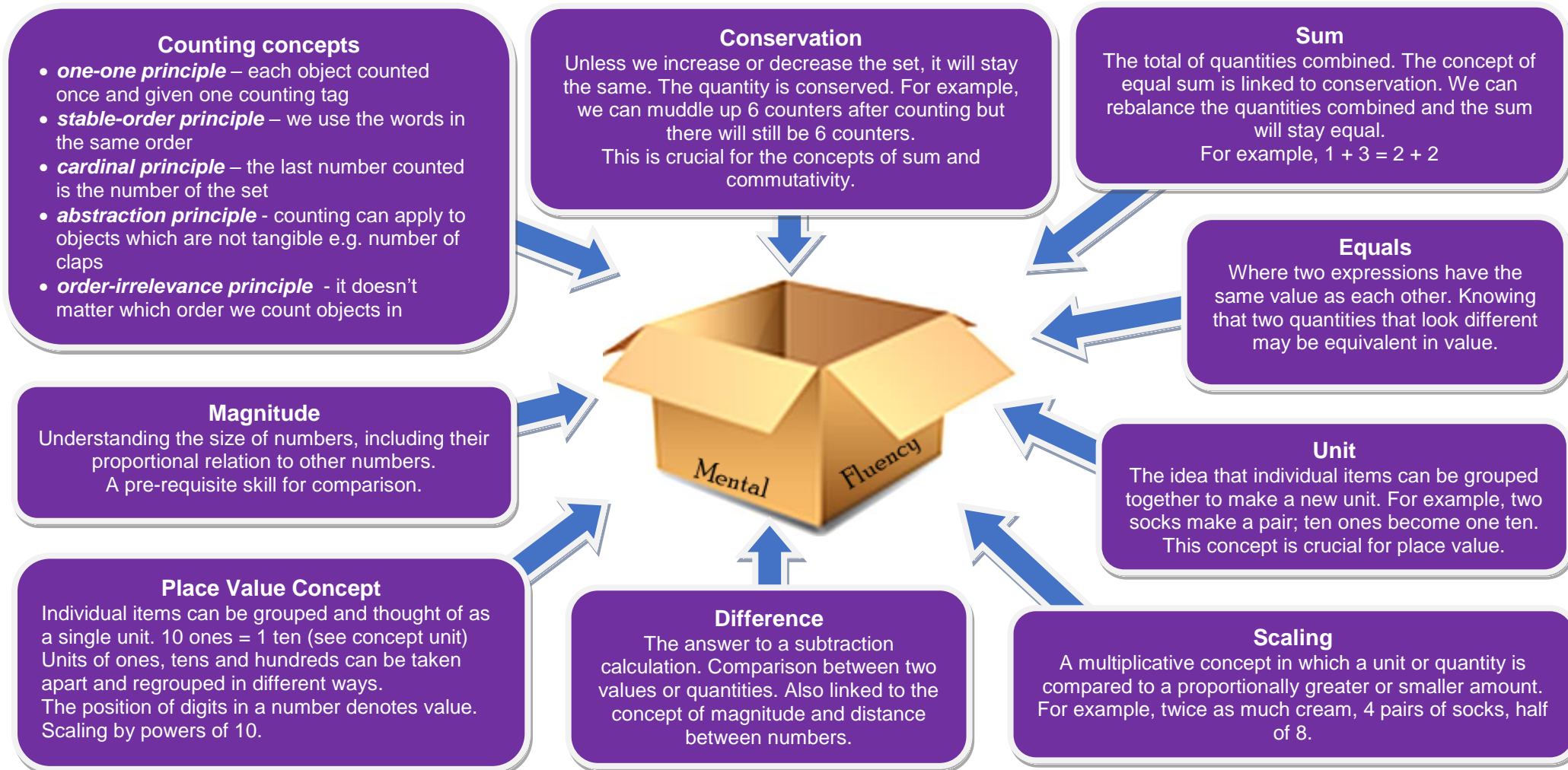
Subitising	the ability to see number as pattern, such as dice patterns. This supports pupils to see numbers within numbers and better regrouping (partitioning).
Regrouping (partitioning)	the ability to break numbers up and recombine them flexibly
Counting on and counting back	in a variety of interval steps
Reordering	knowing when and how to reorder to make calculations easier
Finding complements	links to reordering, identifying useful complements pairs or trios of 1, 10, 60 etc.
Applying the inverse	use of fact family knowledge to 'undo'
Rounding	to a range of benchmark numbers
Estimation	both linear estimation on number lines and scales, and of quantities and calculations to support an increasing sense of what is reasonable
Compensation	to use rounding to add or subtract too much or too little and adjust accordingly
Rebalancing	to adjust the parts of addition and subtraction facts to make a calculation easier
$\times \div$ by powers of 10	
Doubling and halving	
Rearranging	to adjust the groups in multiplication and division to make a calculation easier

• Core concepts

Secure mental fluency is dependent upon a range of underpinning concepts that develop over the primary phase. These are identified in the progression document by a purple box.

Core concept: UNITISING

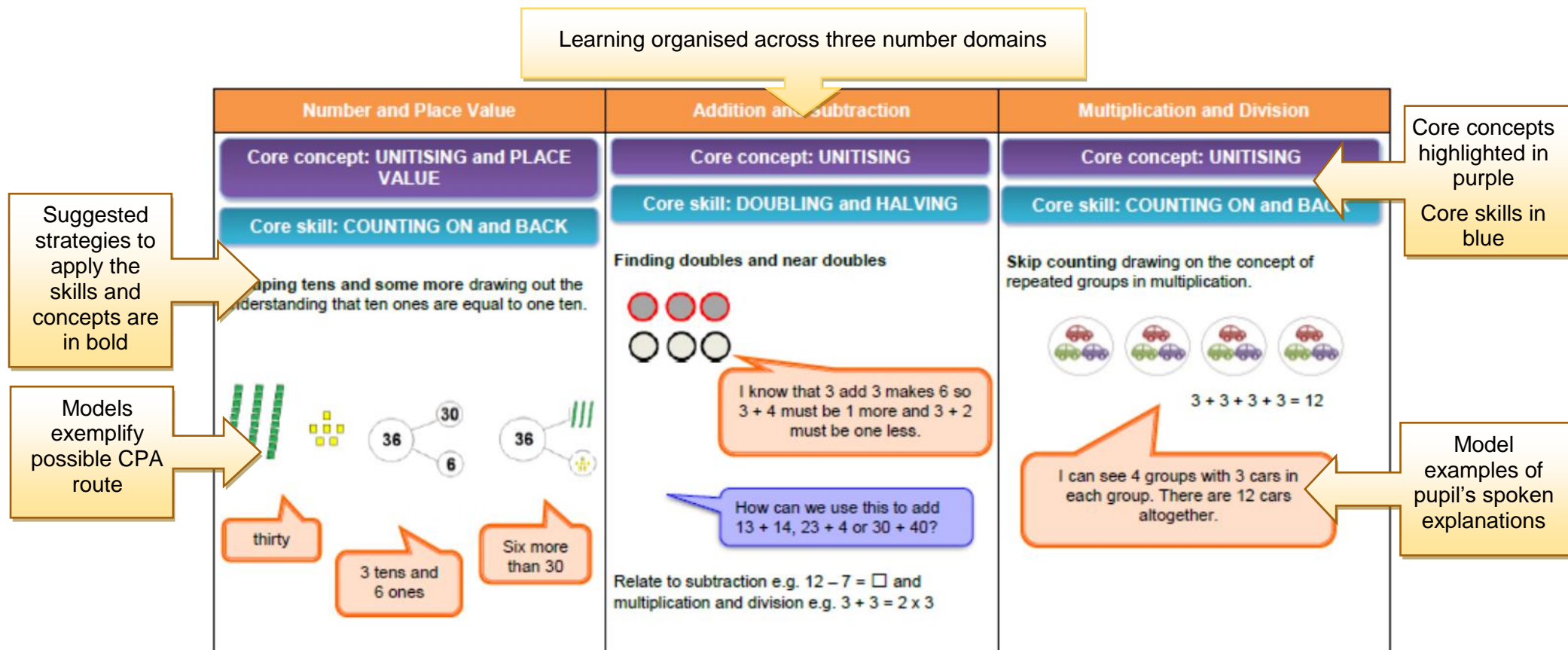
We recommend that schools monitor how these concepts are evident through the school and how they build progressively through each phase.



Using the progression document

The progression is structured into phases. In Years 1 to 4, this is organised into individual year groups. At the beginning, there is a section entitled 'Pre-operational Learning'. This helps ensure that the foundations are secure by the end of EYFS and in the first few weeks in Year 1 before mental fluency within numbers to 10 begins. This also supports the early identification of gaps and barriers.

In each year group / phase, the progression is organised into the National Curriculum Programs of Study domains: number and place value; addition and subtraction; multiplication and division including fractions. Within these domains, key **concepts** (ideas), **skills** (which can be utilised) and **strategies** (methods) are exemplified within the relevant number ranges.



At the end of each phase, a selection of possible examples that align with a given strategy or skill are included. For KS1 and UKS2, there are examples taken directly from the relevant end of key stage assessments (2016) and sample papers. When designing opportunities to practise or for strategy discussions, these will support teachers to explore and / or guide pupils towards a particular strategy.

Upper KS2 examples

<p>Place Value</p> <p>937 + 100 1969 + 100 546 - 40</p> <p>1.7 + 0.05 40 000 - 500</p> <p>246 ÷ 1 100 x 217 0.4 ÷ 10</p> <p>1.68 x 100 100 x 100</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>435 - 30 979 + 100 3.005 + 6.12 2.15 + 0.05</p> <p>100 x 412 0.9 ÷ 10 1.28 x 100 50,000 - 500</p> <p>10 x 100</p> <p>Two decimal numbers add together to equal 1 One of the numbers is 0.007. What is the other number?</p> <p>Circle two numbers that added together make 0.25</p> <p>0.05 0.23 0.2 0.5</p> <p>Circle two numbers that multiply together to equal 1 million</p> <p>200 2,000 5,000 50,000</p> <p>Write the number that is 5 less than 10 million</p> <p>Write the number that is one hundred thousand less than six million</p>	<p>Compensation</p> <p>56 + 8 72 + 9 56 - 8 72 - 9</p> <p>371 + 18 255 + 49 304 + 299</p> <p>673 - 99 854 - 398 3720 - 996</p> <p>0.71 + 0.09 0.56 + 0.08 0.34 - 0.09</p> <p>£1.17 + £0.39 £8.89 - £4.99</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>468 - 9 472 - 9 15.98 + 26.314</p> <p>12 - 6.01 15.4 - 8.88</p> <p>Rebalancing - Equal sum</p> <p>56 + 8 72 + 9 371 + 18 255 + 49</p> <p>304 + 267</p> <p>£37.67 + £3.85 563 + 397 890,488 + 4,890</p> <p>229,899 + 31,321</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>89,994 + 7,643 936 + 285 89,994 + 7,643</p> <p>Rebalancing - Equal difference</p> <p>95 - 10 12 - 17 99 - 12 127 - 102</p>	<p>Think Partition for x and ÷</p> <p>32 x 4 29 x 2 122 x 4 4.6 x 2</p> <p>75 x 3 8.3 x 6 39 x 7</p> <p>3.3 x 7 5 x 49 4 x 198 96 x 0.3</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>15 x 6.1 24 x 3 1.52 x 6 7,505 ÷ 5</p> <p>17 x $1\frac{1}{2}$</p> <p>Make links to doubling and halving</p> <p>50 x 28 86 x 50 500 x 70 18 x 2.5</p> <p>86 x 2.5 160 x 35 500 x 88 1.5 x 6.6</p> <p>0.5 x 120 4.5 x 2.2 15% x 346 75% x 220</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>15% x 440 $\frac{2}{5}$ x 140 24 x 3</p> <p>20% of 1500 95% of 240</p> <p>Multiplying and dividing fractions</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p>
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Implementing the progression

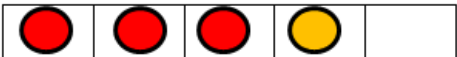

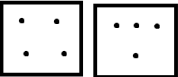

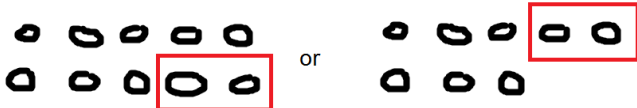

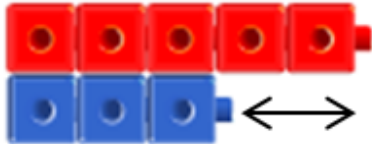
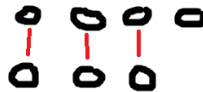
Before implementing the progression, schools should consider some or all of the following self-evaluation questions. These will support leaders to identify the most important focuses and actions.

Is practice effective in your school? How do you know?




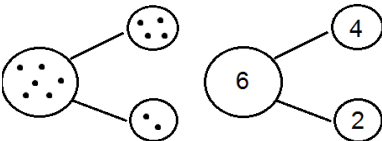

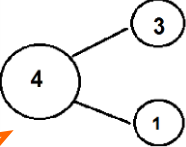

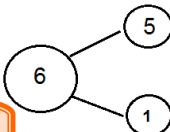

- As a school, are the principles of effective practice design understood?
- Have you identified the key skills and facts in which automaticity for the majority of pupils is the aim?
- Where are the gaps? What are the barriers to pupils developing secure mental fluency?
- To develop effective mental strategies, where does teacher subject knowledge and understanding of approaches need to be strengthened?
- Is there a common language when discussing mental fluency?
- Which strategies did pupils use in the end of key stage assessments? Were these strategies informed by mental fluency?
- What does progression look like now for each of the number domains in mental fluency?

Implementing any new curriculum focus and related approaches should focus upon the impact on pupil outcomes. This includes both quantitative and qualitative measures. Any development of teaching and learning should have this priority at its centre. Actions need to be specific, matched to intended goals and clear to all stakeholders. Implementation should include opportunities for evidence-based reflection points. The emphasis, here, is evaluative and lessons learned should be shared across the community before the next steps are considered.

Pre-operational Learning

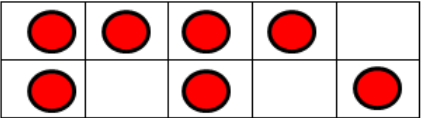
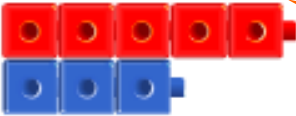
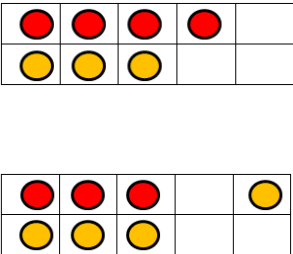
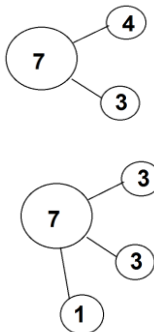



Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core concept: COUNTING</p> <p>Core skill: SUBITISING</p> <p>Numbers to 5 recognising dot patterns on dice / dominoes and fives frames.</p>   <p>Matching patterns where the number of dots is equal. Progress to patterns where the number of dots are equal but the pattern is different.</p>  <p>Finding dot patterns that are one more or one less than the pattern displayed.</p>  <p>I can see 3 dots and 1 dot. There are 4 dots altogether.</p>	<p>Core concept: COMPARISON</p> <p>Comparison model within numbers to 10 drawing out language of greater, more, less, fewer than and equal.</p> <p>Direct comparison</p>  <p>Matching</p>  <p>Discrete cube model</p> 	<p>Core concept: COMPARISON</p> <p>Equal grouping drawing out the concept of equal and unequal amounts.</p> <p>Identifying when groups of objects are equal or unequal using one to one correspondence.</p>  <p>The two groups are unequal.</p>

Pre-operational Learning

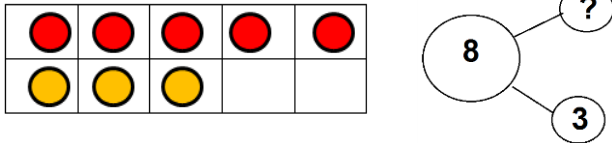
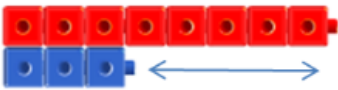

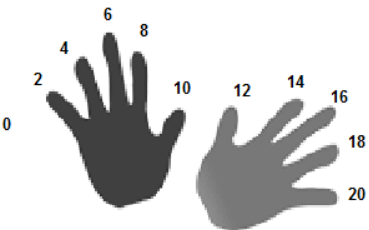
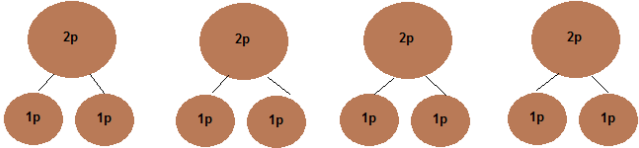
Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core concept: COUNTING</p> <p>Core skill: SUBITISING</p> <p>Numbers 6 and 7 where 5 is the benchmark i.e. 6 is 1 more than 5 using fives frames and additional counters.</p>  <p>Ensure transference to fingers.</p>  <p>7 fingers can be shown by 5 fingers and 2 more fingers.</p> <p>Matching patterns where the number of dots is equal. Progress to patterns where the number of dots are equal but the pattern is different.</p> <p>Finding dot patterns that are one more or one less than the pattern displayed.</p> <p>Identifying numbers within the whole set of dots (i.e. conceptual subitisation).</p> 	<p>Core concept: UNITISING</p> <p>Core skill: REGROUPING</p> <p>Key model: Part whole model drawing out the understanding that in addition and subtraction there is a relationship between parts and a whole.</p>  <p>Four and two make six. Two more than four is six. There is a four and two in six.</p> <p>One more, one less Where one of the parts is 1; signifying one more or 1 less.</p>  <p>One less than 4 is 3. One more than three is four.</p>  <p>3 + 1 = 4 1 + 3 = 4 4 - 1 = 3 4 - 3 = 1</p> <p>Think 5 Where the whole is 6 or 7 and one of the parts is 5 (to secure benchmark from 5).</p>  <p>Six is one more than five. Five is one fewer than six.</p>  <p>6 = 5 + 1 6 = 1 + 5 6 - 1 = 5 6 - 5 = 1</p>	<p>Core concept: UNITISING</p> <p>Patterning drawing out the concept of a repeated unit.</p> <p>Identifying the part of the pattern that repeats (i.e. identifying the unit being repeated).</p>  <p>Move the pupils through the following steps:</p> <ul style="list-style-type: none"> replicate repeated patterns predict the next part of a repeating pattern recognise missing elements of a repeating pattern.

Pre-operational Learning

Year 1

Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core concept: COUNTING and COMPARISON</p> <p>Core skill: SUBITISING</p> <p>Numbers to 10 recognising dot patterns on dice / dominoes and tens frames.</p>  <p>I can see three and three and one makes seven. Four and one and one and one makes seven.</p> <p>Progression Matching patterns where number of dots is equal. Matching patterns where the number of dots is equal but the pattern is arranged differently. Finding dot patterns that are one more or one less than the pattern displayed. Identifying numbers within the whole set of dots (see example above).</p>	<p>Core concept: COMPARISON</p> <p>Core skill: COUNTING ON and BACK</p> <p>Pupils count on to find the total and difference.</p>  <p>Three and two more equals five. Two fewer than five equals three.</p> <p>Core concept: CONSERVATION</p> <p>Core skill: REGROUPING</p> <p>Part part whole model drawing out an understanding of commutativity.</p>   <p>Pupils to extract fact families from the models and explore commutativity.</p>	<p>Core concept: UNITISING</p> <p>Equal grouping drawing out understanding of repeated addition.</p>    <p>and</p> $2 + 2 + 2 = 6$ <p>There are three groups of two teddies. Three groups of two equals six.</p>

Year 1
Numbers to 10

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Core skill: APPLYING THE INVERSE</p> <p>Think addition to solve subtraction – leading to greater understanding that if we know one part we can use that to find the unknown part.</p> <p>8 - 3</p>   <p>I can see that eight can be split into three and five.</p> <p>I know that five and three are eight.</p> <p>I can see that five more than three is eight.</p>	<p>Core skill: COUNTING ON and BACK</p> <p>Skip counting Counting groups of objects with two hands (drawing out understanding of doubles).</p>  <p>To include opportunities to count in 2s in several ways.</p>  

Year 1
Numbers to 10

Number and Place Value

Addition and Subtraction

Multiplication and Division

Core concept: CONSERVATION

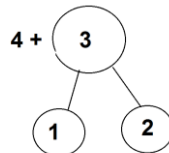
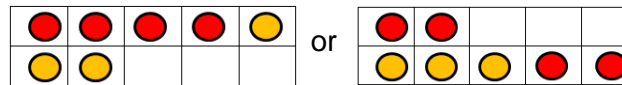
Core skill: REGROUPING

Think 5 for addition using five as a benchmark number.

$$4 + 3 =$$



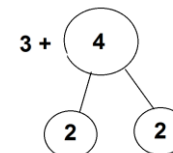
can become



$$4 + 1 + 2 =$$

$$5 + 2 =$$

I know that four and one more is five.
I can see that three can be split into one and two.
Five and two more is seven.

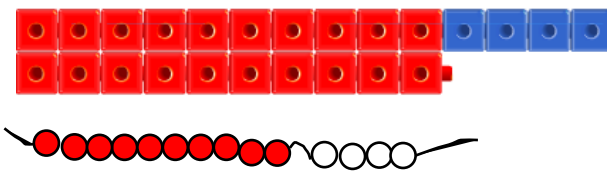
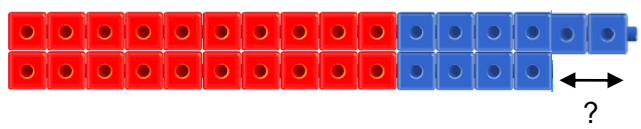
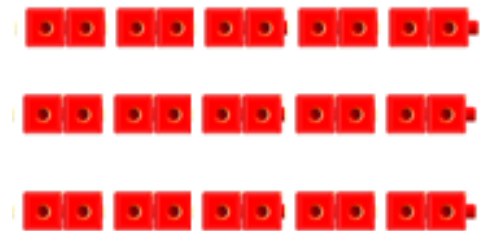


$$3 + 2 + 2 =$$

$$5 + 2 =$$

I know that three and two more is five.
I can see that four can be split into two and two.
Five and two more is equal to seven.

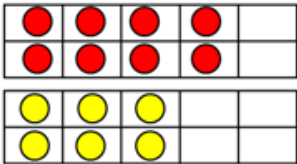
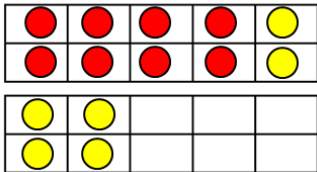
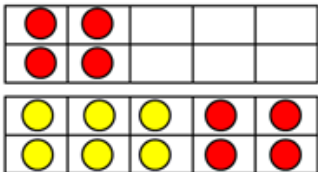
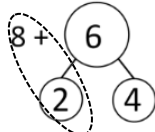
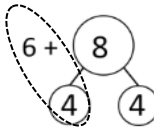
Year 1
Numbers to 10

Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core concept: COMPARISON and PLACE VALUE</p> <p>Comparing numbers using ten as a benchmark number.</p>  <div> $14 - 4 = 10$ $4 + 10 = 14$ $10 = 14 - 4$ $4 = 14 - 10$ </div> <div> <p>14 is 4 more than 10. 4 more than 10 is 14. 10 is 4 fewer than 14. 10 fewer than 14 is 4.</p> </div> <p>Explore the language of 'more than' and 'less than' through measures and bar charts.</p>	<p>Core concept: COMPARISON</p> <p>Core skill: COUNTING ON and BACK</p> <p>Comparison model Pupils to count on to find total and difference.</p>  <div> $14 + \square = 16$ $\square + 14 = 16$ $16 - 14 = \square$ $16 - \square = 14$ </div> <div> <p>14 and 2 more equals 16. 2 fewer than 16 equals 14.</p> </div>	<p>Core concept: UNITISING</p> <p>Equal grouping drawing out the concept of repeated addition.</p>  <p>$10 + 10 + 10$ 3 groups of 10 equals 30</p> <div> <p>I can see 10 and 10 and 10 makes 30. 3 equal groups of 10 makes 30. 30 can be split into 3 groups of 10.</p> </div>

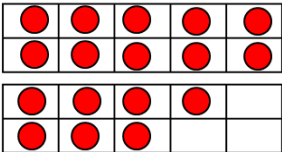
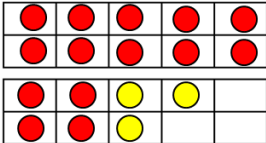
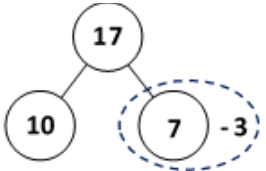
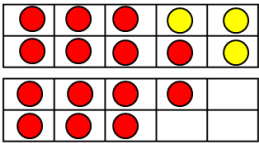
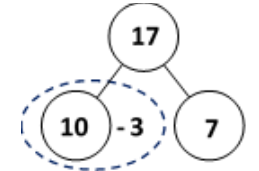
Year 1
Numbers to 20

Number and Place Value	Addition and Subtraction	Multiplication and Division
<div data-bbox="103 165 763 272" data-label="Text"> <p>Core concept: UNITISING and PLACE VALUE</p> </div> <div data-bbox="103 284 763 352" data-label="Text"> <p>Core skill: REGROUPING</p> </div> <p data-bbox="103 384 763 480">Regrouping numbers into ten and some more drawing out understanding that ten ones are equal to one ten.</p> <div data-bbox="136 531 714 722" data-label="Figure"> </div> <div data-bbox="159 743 705 1034" data-label="Text"> <p>I made 14 into 10 and 4 more. There is 1 ten and 4 ones in the number 14. Here is the ten and here are the 4 more.</p> </div>	<div data-bbox="786 165 1447 234" data-label="Text"> <p>Core skill: APPLYING THE INVERSE</p> </div> <p data-bbox="786 280 1447 376">Think addition to solve subtraction using the comparison or part whole model to identify the missing part.</p> <p data-bbox="786 400 931 432">$16 - 2 = \square$</p> <div data-bbox="826 472 1429 663" data-label="Figure"> </div> <div data-bbox="801 695 1099 986" data-label="Text"> <p>I can see that sixteen can be split into fourteen and two.</p> </div> <div data-bbox="1120 679 1451 995" data-label="Text"> <p>I know that fourteen and two more is sixteen. So 16 subtract two is fourteen.</p> </div>	<div data-bbox="1491 165 2130 234" data-label="Text"> <p>Core skill: COUNTING ON and BACK</p> </div> <p data-bbox="1491 280 2130 416">Skip counting Counting to include opportunities to count in 5s and 10s in several ways including with coins, tallies and pictograms.</p> <div data-bbox="1603 456 1984 655" data-label="Figure"> </div>

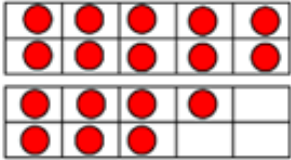
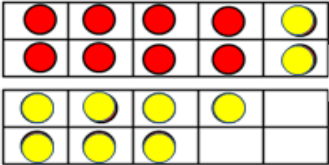
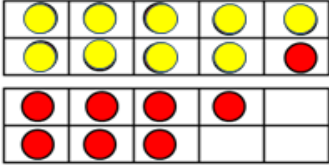
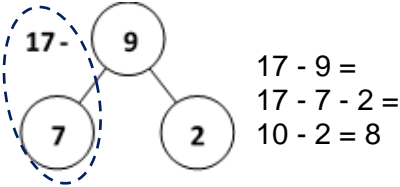
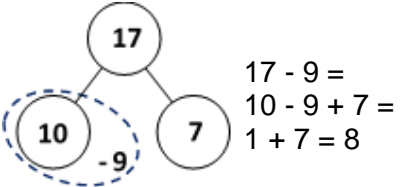
Year 1
Numbers to 20

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<div>Core concept: CONSERVATION</div> <div>Core skill: REGROUPING</div> <p>Regrouping numbers to 20 leading to 'think 10 for addition'. Pupils should experience regrouping either addend.</p> <div> $8 + 6$  </div> <p>can become</p> <div>  <p>or</p>  </div> <div> $8 + 6$  $8 + 2 + 4 =$ $10 + 4 =$ </div> <div> $6 + 8$  $6 + 4 + 4 =$ $10 + 4 =$ </div> <div> <p>I know that eight and two more is ten. Four and two make six. So $8 + 6$ can become $8 + 2 + 4$.</p> </div> <div> <p>I know that six and four more is ten. Four and four make ten. So $6 + 8$ can become $6 + 4 + 4$.</p> </div>	

Year 1
Numbers to 20

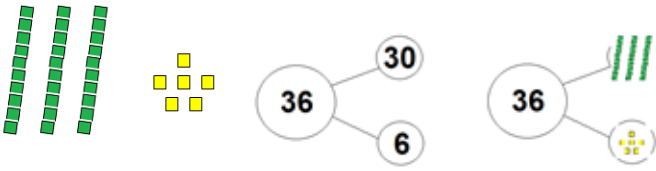
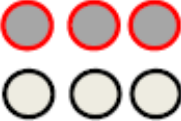

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>'Think 10 for subtraction' using the minuend <i>or</i> the subtrahend. First with numbers where no bridging through ten is required.</p> <p style="text-align: center;">$17 - 3$</p>  <p>Regrouping the minuend in two ways and recombining the remaining quantity.</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;">   $17 - 3 =$ $7 - 3 + 10 =$ $4 + 10 = 14$ </div> <div style="text-align: center;"> <p>or</p>   $17 - 3 =$ $10 - 3 + 7 =$ $7 + 7 = 14$ </div> </div> <div style="border: 2px solid orange; border-radius: 15px; padding: 10px; margin-top: 20px; text-align: center;"> <p>I know that 17 can be regrouped into 10 and 7. I can take 3 from either 10 or 7.</p> </div>	

Year 1
Numbers to 20

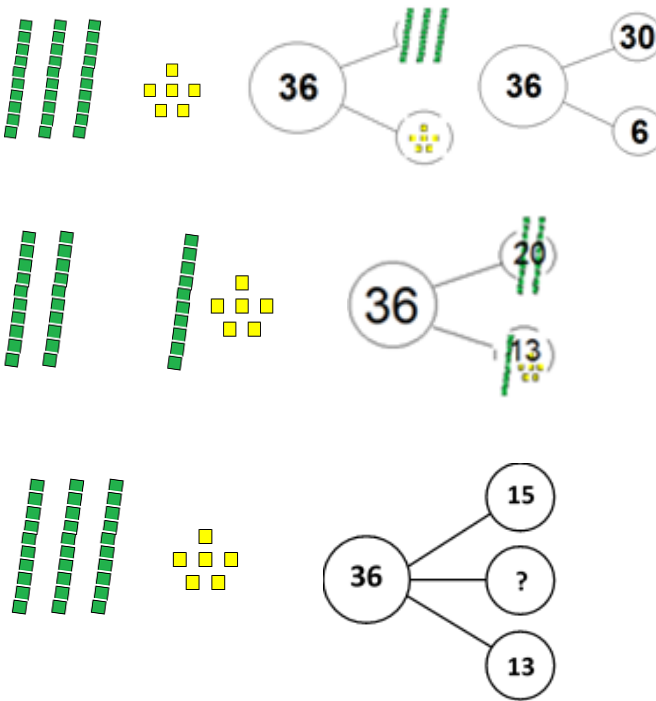
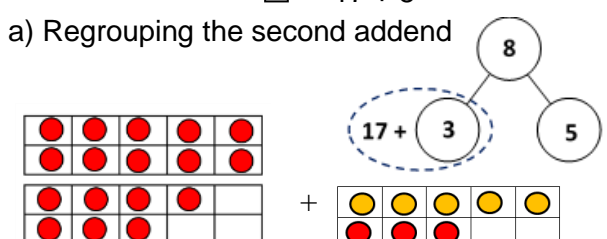
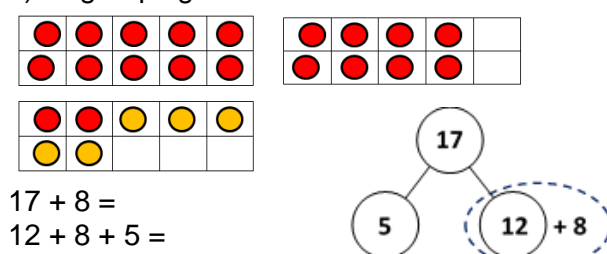
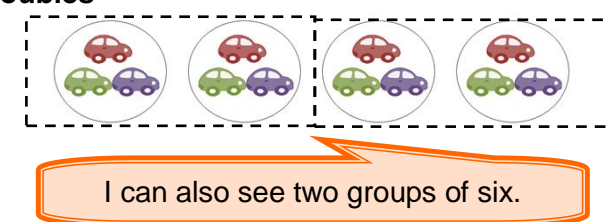
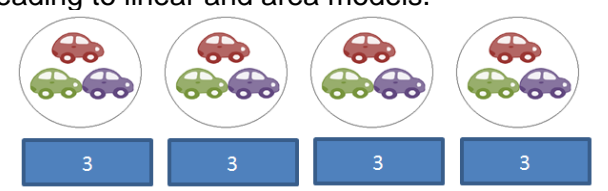
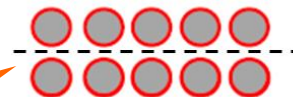
Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Then with numbers which would require bridging through ten.</p> <p>For example, $17 - 9$</p>  <p>Regrouping the subtrahend</p>  <p>or</p> <p>Regrouping the minuend</p>  <p>  $17 - 9 = 8$ $17 - 7 - 2 = 8$ $10 - 2 = 8$ </p> <p>  $17 - 9 = 8$ $10 - 9 + 7 = 8$ $1 + 7 = 8$ </p> <div data-bbox="701 887 1088 1222"> <p>Nine can be regrouped into 7 and 2. I can take 7 from 17 to leave 10 and then I can use my number bonds to take away 2 more.</p> </div> <div data-bbox="1099 887 1520 1222"> <p>Seventeen can be regrouped into 10 and 7. Then I can use my number bonds to take 9 from 10. I'm left with 1. Then I add one to seven.</p> </div>	

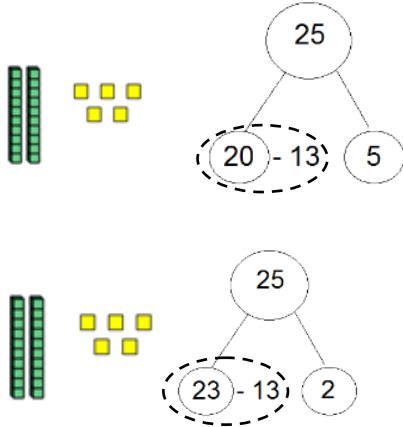
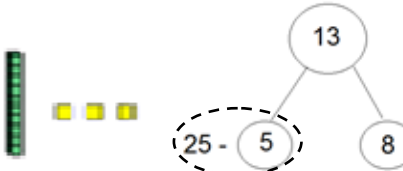
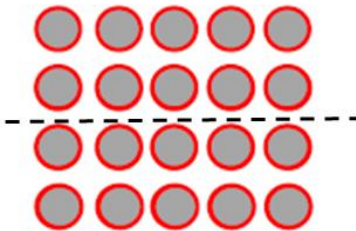
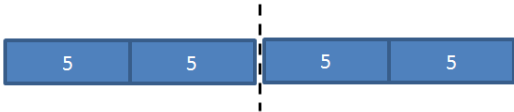
Year 1
Numbers to 20

Year 2

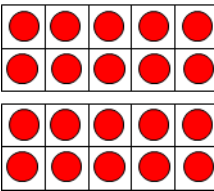
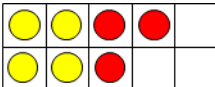
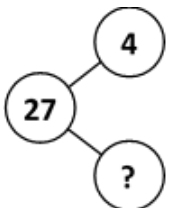
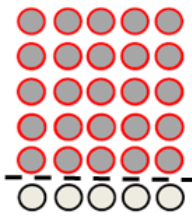
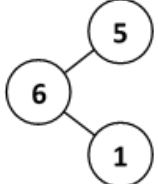


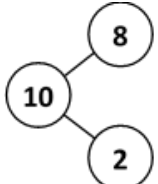

Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core concept: UNITISING and PLACE VALUE</p> <p>Core skill: COUNTING ON and BACK</p> <p>Grouping tens and some more drawing out the understanding that ten ones are equal to one ten.</p>  <p>thirty-six</p> <p>3 tens and 6 ones</p> <p>6 more than 30</p>	<p>Core concept: UNITISING</p> <p>Core skill: DOUBLING and HALVING</p> <p>Finding doubles and near doubles</p>  <p>I know that 3 add 3 makes 6. So $3 + 4$ must be 1 more. $3 + 2$ must be one less.</p> <p>How can we use this to add $13 + 14$, $23 + 4$ or $30 + 40$?</p> <p>Relate to subtraction e.g. $12 - 7 = \square$ and multiplication and division e.g. $3 + 3 = 2 \times 3$.</p>	<p>Core concept: UNITISING</p> <p>Core skill: COUNTING ON and BACK</p> <p>Skip counting drawing on the concept of repeated groups in multiplication.</p>  <p>$3 + 3 + 3 + 3 = 12$</p> <p>I can see 4 groups with 3 cars in each group. There are 12 cars altogether.</p>

Year 2
Numbers to 100

Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core concept: CONSERVATION and PLACE VALUE</p> <p>Core skill: REGROUPING</p> <p>Regroup 2-digit numbers flexibly and in multiple ways.</p> 	<p>Core concept: CONSERVATION</p> <p>Core skill: REGROUPING</p> <p>Think 10 for addition (Tens Ones + Ones)</p> <p>Drawing out the skill of regrouping numbers to allow bridging through tens: Tens Ones + Ones. Exploring that either addend can be regrouped and utilise benchmark numbers.</p> <p>$\square = 17 + 8$</p> <p>a) Regrouping the second addend</p>  <p>$17 + 8 =$ $17 + 3 + 5 =$ $20 + 5 = 25$</p> <p>b) Regrouping the first addend</p>  <p>$17 + 8 =$ $12 + 8 + 5 =$ $20 + 5 = 25$</p>	<p>Core concept: UNITISING and SCALING</p> <p>Core skill: DOUBLING and HALVING</p> <p>Doubles</p>  <p>I can also see two groups of six.</p> <p>Leading to linear and area models.</p>  <p>3 3 3 3</p> <p>There are 4 groups of 3 cars. That is 12 cars altogether. I can see 12 cars with 4 groups of 3 cars.</p>  <p>2 x 5 is equal to double 1 x 5.</p> <p>Year 2 Numbers to 100</p>

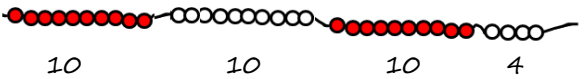
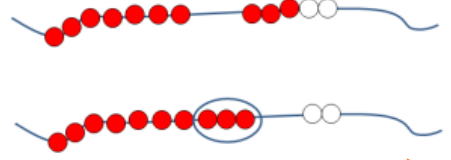
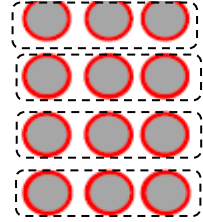

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Think 10 for subtraction (Tens Ones - Ones) Exploring that either the minuend or the subtrahend can be regrouped.</p> <p>$25 - 13 = \square$</p> <p>Regrouping the minuend (two examples). Taking from a multiple of ten or taking to a multiple of ten.</p> <div data-bbox="808 459 1404 887">  <div data-bbox="1256 491 1404 592"> $20 - 13 + 5$ $= 7 + 5$ $= 12$ </div> <div data-bbox="1256 715 1404 815"> $23 - 13 + 2$ $= 10 + 2$ $= 12$ </div> </div> <p>Regrouping the subtrahend – normally to a multiple of ten.</p> <div data-bbox="808 1038 1404 1209">  <div data-bbox="1279 1066 1404 1166"> $25 - 5 - 8$ $= 20 - 8$ $= 12$ </div> </div>	  <div data-bbox="1615 639 2085 863"> <p>4 x 5 is double 2 x 5. I can show it as an array and as a linear model.</p> </div>

Year 2
Numbers to 100

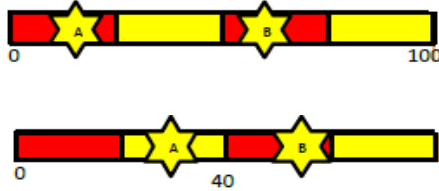



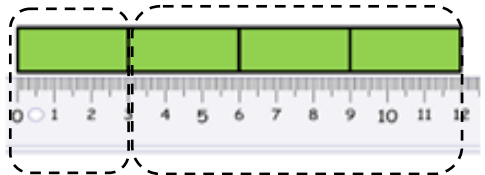
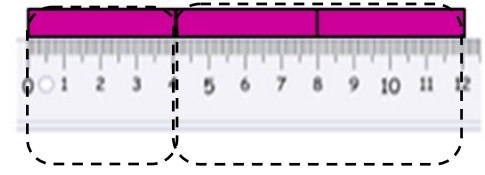
Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Core concept: UNITISING</p> <p>Core skill: FINDING COMPLEMENTS / REORDERING</p> <p>Adding three, one-digit numbers such as $7 + 5 + 6$ or $6 + 7 + 4$ and drawing out the reasons why pupils may wish to reorder the numbers. Focus upon a range of strategies used.</p> <p>Core skill: APPLYING THE INVERSE</p> <p>Think addition to solve subtraction</p> <p>Tens Ones - Ones = <input type="text"/> without regrouping</p> <div>   </div> <div>  <div> $27 - \square = 4$ $4 + \square = 27$ </div> </div> <p>I know 4 and 3 makes 7 so 4 and 23 make 27.</p>	<p>Core concept: CONSERVATION</p> <p>Core skill: REGROUPING</p> <p>Applying understanding of benchmark numbers.</p> <p>Think 5 for multiplication and division</p> <p>$6 \times 5 = 5 \times 5 + 5 \times 1$</p> <div>   <p>5 x 5 and one more group of 5 is equal to 6 x 5.</p>  </div> <p>Think 10 for multiplication and division</p> <p>$8 \times 5 = 10 \times 5 - 2 \times 5$</p> <div>   <p>8 x 5 is two groups of 5 fewer than 10 x 5.</p>  </div>

Year 2
Numbers to 100

Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core concept: COUNTING and PLACE VALUE</p> <p>Core skill: COUNTING ON and BACK</p> <p>Counting on and back in ones and tens from any number allowing children to notice in the pattern what changes as a result (and what doesn't change).</p> <p>34 3 tens and 4 ones</p> <p>44 4 tens and 4 ones</p> <p>46 4 tens and 6 ones</p> <p>36 3 tens and 6 ones</p> <p>26 2 tens and 6 ones</p> <p>I can see that the tens are changing but the ones are staying the same.</p> <p>Then counting with coins and on scales from any amount.</p>	<p>Core concept: COMPARISON</p> <p>Core skill: COUNTING ON and BACK</p> <p>Drawing out complements to benchmark numbers.</p> <p>25 13 +7 +5</p> <p>I know there is 7 more to 20 and 5 more to 25.</p> <p>Core skill: COMPENSATION</p> <p>Compensation at this stage is a form of Think 10, utilising benchmark numbers.</p> <p>$17 + 8 = \square$</p> <p>17 + 10 subtract 2</p> <p>8 2</p> <p>Adding 8 is like adding ten and taking 2 away. Subtracting 8 is like subtracting ten and adding 2 back.</p> <p>Apply this to subtraction.</p>	<p>Core skill: APPLYING THE INVERSE</p> <p>Think multiplication Introduction of the array and linear model to explore how the relationship of multiplication and division relate.</p> <p>In multiplication, explore how multiplier, multiplicand and product interrelate.</p> <p>In division, explore how dividend, divisor and quotient interrelate and link to multiplication.</p> <p>$4 \times 3 = 12$</p> <p>Number in each group 3</p> <p>Number of groups 4</p> <p>12</p> <p>$4 \times 3 = 12$ $3 \times 4 = 12$ $12 \div 3 = 4$ $12 \div 4 = 3$</p> <p>Year 2 Numbers to 100</p>

Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core concept: COMPARISON and PLACE VALUE</p> <p>Comparing numbers using tens as benchmark numbers.</p>  <p> $34 - 4 = 30$ $4 + 30 = 34$ $30 = 34 - 4$ $4 = 34 - 30$ </p> <p>34 is 4 more than 30. 4 more than 30 is 34. 30 is 4 fewer than 34. 30 fewer than 34 is 4.</p> <p>Explore the language of 'more than' and 'less than' through measures and bar charts.</p>	<p>Core concept: CONSERVATION and SUM</p> <p>Core skill: REBALANCING</p> <p>Equal Sum drawing out understanding that the sum remains equal when we rebalance the addends in an addition calculation.</p>  <p>I can prove that $7 + 5 = 10 + 2$ using a bead string.</p>	<p>...continued</p> <p>$12 \div 3 = 4$</p>  <p>I can see that 12 can be shared into 4 equal groups with 3 in each group.</p>  <p>I can see that 3 can be taken from 12, four times.</p> <p>I know that I can use $4 \times 3 = 12$ to answer $12 \div 3$ or $12 \div 4$.</p>

Year 2
Numbers to 100

Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core concept: MAGNITUDE</p> <p>Core skill: ESTIMATION</p>  <p>Drawing out understanding of the distance of numbers to target numbers.</p>  <p>24 is 4 away from 20 and 6 away from 30. 24 is nearer to 20 than to 30.</p>	<p>Core concept: COMPARISON</p> <p>Core skill: REBALANCING</p> <p>Equal Difference drawing out understanding that adding or subtracting the same quantity from both the subtrahend and minuend maintains the difference between the numbers.</p> <p>5 - 3 is equal to 7 - 5</p>  <p>5 - 3 is equal to 3 - 1</p> 	<p>...continued</p> <p>Leading to fractional understanding</p>  <p>I can see a quarter of 12 is equal to 3 and three quarters of 12 is equal to 9.</p>  <p>One third of 12 equals 3. Two thirds of 12 is 9.</p>

Year 2
Numbers to 100

Key Stage 1 Examples

'Think 10' Regroup

$$\begin{array}{cccccc} 5+6 & 7+4 & 9+7 & 7+6 & 8+7 & 7+5 \\ 2+18 & 4+18 & 8+19 & 47+6 & 68+7 & 9+87 \\ 13-8 & 27-8 & 53-6 & 68-\square=7 & 73+\square=89 \end{array}$$

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

$$\begin{array}{cccc} 8+6=\square & 5+7=\square & 12-7=\square & 46+7=\square \\ 8+5+4=\square & 55+17=\square & 71-14=\square & \\ 86-21=\square & 65+\square=93 & & \end{array}$$

'Think Addition' for subtraction

$$8-5 \quad 9-6 \quad 6-2 \quad 80-50 \quad 19-6 \quad 60-20$$

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

$$\begin{array}{cccc} 10-\square=2 & \square+5=9 & 12-7=\square & 19-9=\square \\ 17-6=\square & 39-8=\square & 50-\square=20 & 56-\square=51 \end{array}$$

Reordering and finding complements

$$\begin{array}{cccc} 5+4+5 & 2+3+8 & 2+4+6 & 6+3+7 \\ 36+5+4 & 54+26 & & \end{array}$$

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

$$4+5+6=\square \quad 36+24=\square \quad 69+11=\square$$

Compensation

$$\begin{array}{cccccc} 2+9 & 12+9 & 9+72 & 2+19 & 19+42 & 42+39 \\ 5+8 & 15+8 & 65+8 & 18+5 & 55+18 & 48+35 \\ 12-9 & 22-9 & 52-9 & 52-19 & 92-19 & 92-39 \\ 12-8 & 22-8 & 52-8 & 52-18 & 92-18 & 92-48 \\ 48+\square=92 & 8+\square=52 & & & & \end{array}$$

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

$$\begin{array}{cccc} 8+6=\square & 52+7=\square & 28+\square=35 & 69+11=\square \\ 55+17=\square & 39-8=\square & 43+38=\square & 70-18=\square \end{array}$$

Rebalancing - Equal sum

$$\begin{array}{cccccc} 12+9 & 9+72 & 24+19 & 15+42 & 44+37 & \\ 5+8 & 15+8 & 65+7 & 18+6 & 55+15 & \\ 48+35 & & & & & \end{array}$$

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

$$\begin{array}{ccc} 8+6=\square & 4+5+6=\square & 69+11=\square \\ 55+17=\square & 36+24=\square & 43+38=\square \\ 8+5+4=\square & & \end{array}$$

Rebalancing - Equal difference

$$\begin{array}{cccccc} 32-7 & 25-8 & 55-7 & 55-17 & 92-19 & \\ 97-43 & 48+\square=92 & 8+\square=55 & & & \end{array}$$

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

$$\begin{array}{ccc} 12-7=\square & 28+\square=35 & 71-14=\square \\ 39-8=\square & 86-21=\square & 70-18=\square \\ 65+\square=93 & & \end{array}$$

Reordering and multi-strategy

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

$$8+5+4=\square$$

Double and near double facts

$$\begin{array}{cccccc} 3+3 & 30+30 & 32+32 & 3+4 & 30+40 & \\ 6-3 & 60-30 & 64-32 & 7-3 & 70-40 & \\ 6-\square=3 & 60-\square\square=30 & 64-\square\square= & & & \\ \square\square-3=64 & & & & & \end{array}$$

Find two ways of solving this: $70-\square0=\square0$

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

$$12-7=\square \quad \square+5=9 \quad 50-\square=20$$

Make links to doubling and halving

$$\begin{array}{ccc} 3+3=2 \times 3 & 2 \times 30 & 2 \times 3+1 \\ 6 \div 2 & 60 \div 2 & \end{array}$$

Ensure that pupils can halve odd multiples of ten
 $50 \div 2 =$

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper

$$\begin{array}{ccc} 3 \times 2=\square & 2 \times 0=\square & \\ 8 \div 2=\square & \frac{1}{2} \text{ of } 16=\square & \frac{1}{2} \text{ of } 30=\square \\ 12 \div 2=\square & & \end{array}$$

Key facts



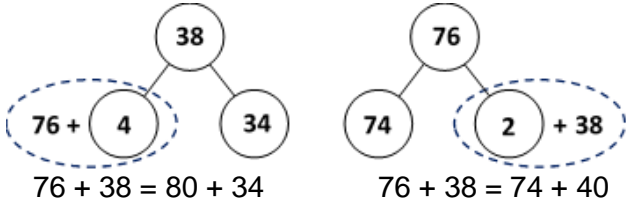
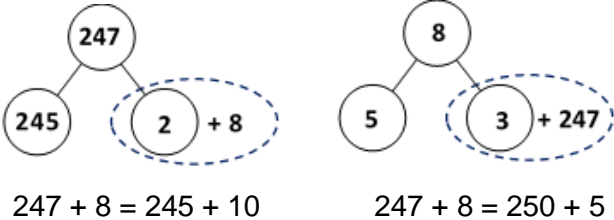
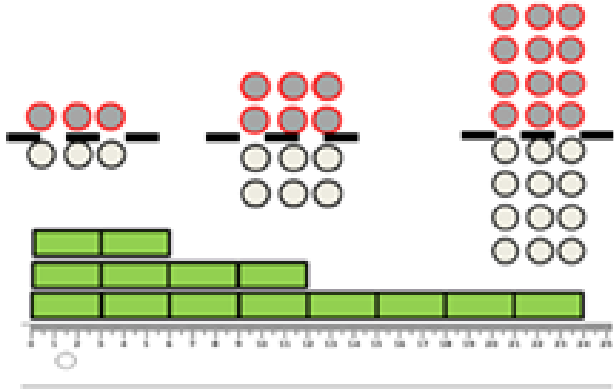
Year One Recall

- Number bonds within 10 including $a + b + c = d$, the effect of adding zero and missing number calculations
- Reordering to find tens and some more e.g. $4 + 5 + 6 =$
- Doubles within 10 including subtraction e.g. $6 - 3 = 3$ and missing numbers e.g. $6 - \square = 3$
- Structured subitisation on tens frame to 20

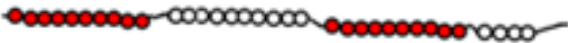
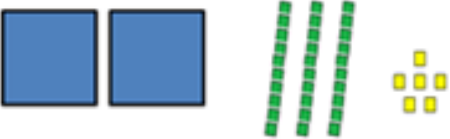
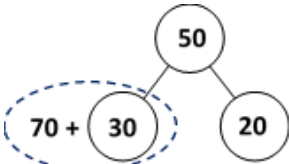
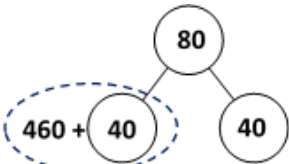
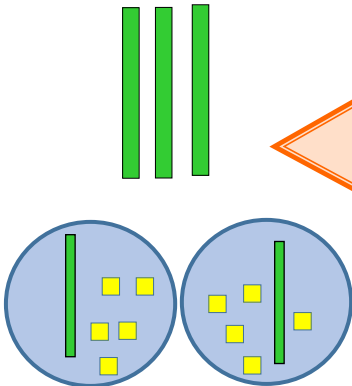
Year Two Recall

- Addition and subtraction facts to 20
- Multiplication and division facts 2, 5 and 10 x tables
- Multiplication facts for 3 x tables
- Number of minutes in an hour; number of hours in a day
- Coin recognition up to £2
- Doubles to 20

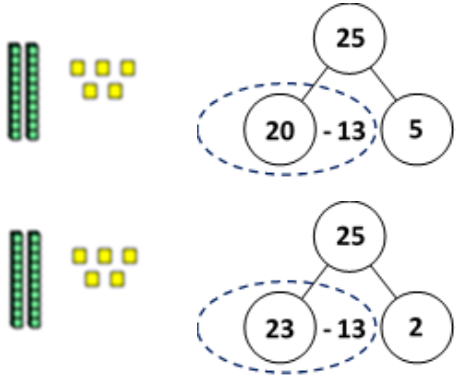
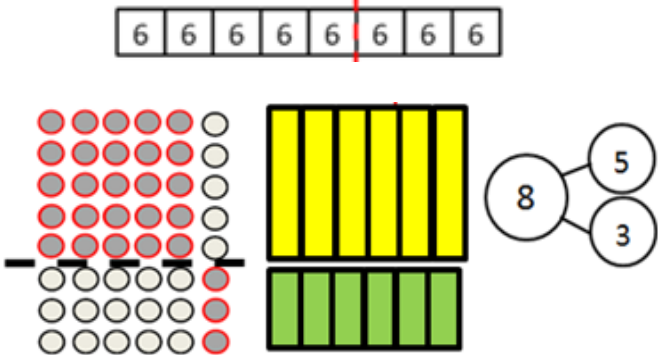
Year 3

Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core concept: UNITISING and PLACE VALUE</p> <p>Core skill: REGROUPING</p> <p>Grouping hundreds, tens and ones drawing out the concept that ten ones are equal to a unit of one ten and that ten tens are equal to a unit of one hundred.</p>  <p>I can see 2 hundreds, 3 tens and 6 ones $200 + 30 + 6$. 236 is also six more than two hundred and thirty.</p> <p>Using part part whole models, regroup 3-digit integers flexibly and in multiple ways.</p>  <p>236 can be regrouped into 220 and 16. There are 23 tens and 6 ones in 236.</p>	<p>Core concept: CONSERVATION</p> <p>Core skill: REGROUPING</p> <p>Think regroup for addition Using part whole models draw out the skill of regrouping numbers to allow bridging through multiples of ten and a hundred. Ask pupils to reason why they may wish to reorder the numbers.</p> <p>Pupils should be encouraged to explore multiple ways of regrouping both addends (refer to number and place value experiences). Only a limited example is shown here e.g. $76 + 38$.</p>  <p>$76 + 38 = 80 + 34$ $76 + 38 = 74 + 40$</p> <p>This can be applied to regrouping addends in 3-digit + 1-digit calculations e.g. $247 + 8$.</p>  <p>$247 + 8 = 245 + 10$ $247 + 8 = 250 + 5$</p>	<p>Core concept: UNITISING and SCALING</p> <p>Core skill: DOUBLING and HALVING</p> <p>To include 'double and double' strategy for x8 and halving strategy for finding x5.</p>  <p>I can find 5 lots by finding 10 lots and halving the product.</p>


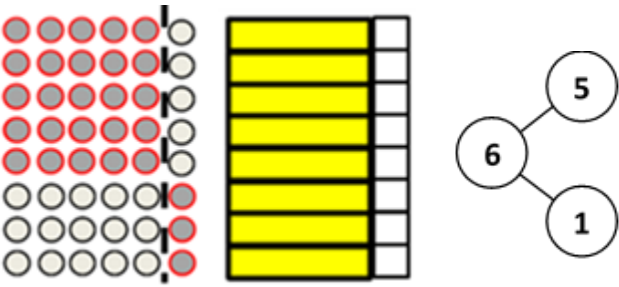
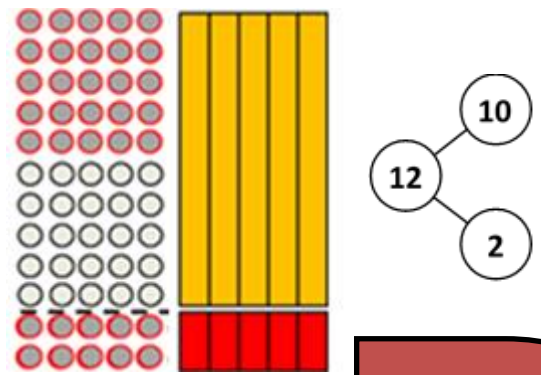
Year 3
Numbers to 1000

Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Repeat this understanding to explore tenths through the same concrete and pictorial representations securing the multiplicative relationship.</p>  <p>3 ones and 4 tenths.</p>  <p>2 tens, 3 ones and 6 tenths.</p>	<p>This can also be applied to regroup to bridge through multiples of 100 e.g. $70 + 50$ or $460 + 80$.</p>  <p>$70 + 50 = 100 + 20$</p>  <p>$460 + 80 = 500 + 40$</p> <p>This sum is easier if I regroup the 80 into 40 and add it to the 460 to make 500.</p>	<p>Ensure pupils can double and halve 2-digit numbers and generalise what happens when we halve a number with an odd multiple of tens.</p>  <p>If I halve 30, I can share one ten into each group but then I have to regroup the last ten into ten ones. Each group will then get five ones. So 1 ten and 5 ones in each group is 15.</p> <p>Extend to halving odd multiples of 100 and later odd ones. Pupils should also be able to use doubles knowledge to solve near double questions e.g. $70 + 60$.</p> <p>$70 + 60$ is like double 60 plus 10. It's also 10 less than double 70.</p>

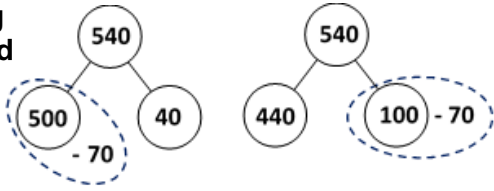
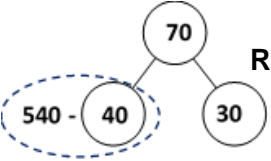
Year 3
Numbers to 1000

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Core concept: UNITISING</p> <p>Core skill: REGROUPING</p> <p>Think Regroup for subtraction drawing out the skill of regrouping either the minuend or subtrahend including when bridging through multiples of ten and a hundred e.g. $25 - 13$.</p> <p>Regrouping the minuend</p> <p>I regrouped the 25 into 20 and 5. Then I took the 13 from the 20 and that left 7. Now I add that to the 5 which makes 12.</p>  <p>I regrouped the 25 into 23 and 2. Then I took the 13 from the 23 and that left 10. Now I add that to the 2 which makes 12.</p>	<p>Core concept: CONSERVATION</p> <p>Core skill: REGROUPING</p> <p>Think 5x fact Application of the distributive law.</p> <p>Regrouping the multiplier (number of groups) e.g. 8×6.</p>  <p>$8 \times 6 = 5 \times 6 + 3 \times 6$</p> <p>I can use my 5 fact to solve 8×6. I know 5×6 is 30 and 3×6 is 18. So 8×6 is 48.</p>

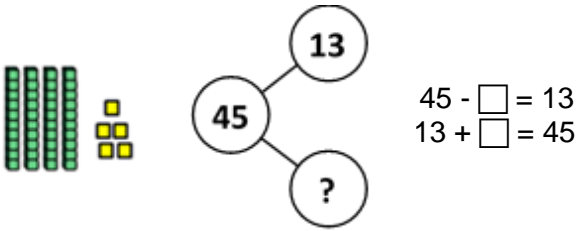
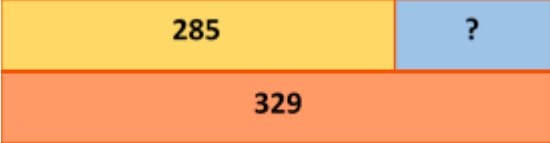
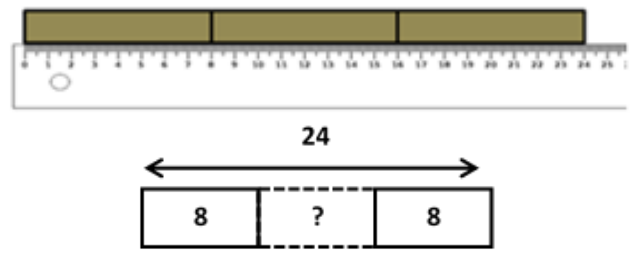
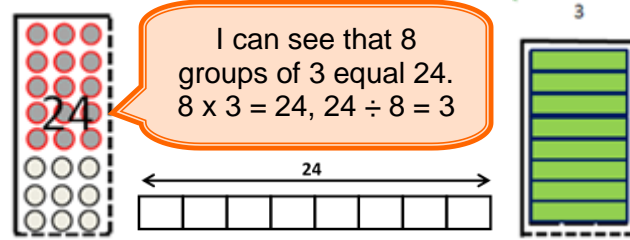
Year 3
Numbers to 1000

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Regrouping the subtrahend</p>  <p>I regrouped the 13 into 5 and 8. Then I took the 5 from the 25 and that left 20. Now I can take away the remaining 8 to make 12.</p>	<p>Regrouping the multiplicand (number in the group) e.g. 8×6.</p>  <p>$8 \times 6 = 5 \times 8 + 1 \times 8$</p> <p>I can use my 5 fact to solve 8×6. I know 8×5 is 40 and 8×1 is 8. So 8×6 is 48.</p> <p>Applying to think 10. For example, 12×5.</p>  <p>$12 \times 5 = 10 \times 5 + 2 \times 5$</p>









Year 3
Numbers to 1000

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Then applied to HTO - O and HTO - TO. For example, $540 - 70$.</p> <p>Regrouping the minuend</p>  <p>Regrouping the subtrahend</p>  <p>Apply to contexts of measures such as money and time e.g. £3 and 40p subtract 60p.</p> <p>I can regroup the 60p into 40p and 20p. First, I take the 40p away. That gets me to £3. Next, I take the 20p away, which is £2 and 80p.</p> <p>Core concept: UNITISING</p> <p>Core skill: FINDING COMPLEMENTS / REORDERING</p> <p>Reordering and finding complements Adding three or more numbers. Draw out reasons why children may wish to reorder the numbers. Focus upon the range of strategies used.</p> $6 + 9 + 4 + 5 + 1 =$ $75 + 95 + 25 =$ $1.5 + 3 + 0.5 =$	

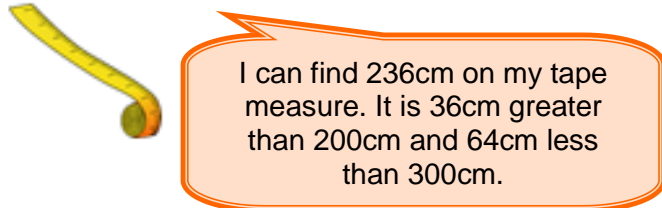


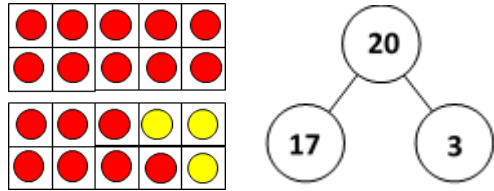

Year 3
Numbers to 1000

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Core skill: APPLYING THE INVERSE</p> <p>Think addition for subtraction</p>  <p>Thirteen and what makes 45?</p>  <p>$329 - 285 = \square$ so $285 + \square = 329$</p>	<p>Core skill: APPLYING THE INVERSE</p> <p>Think multiplication for division e.g. $24 \div 8$</p> <p>By grouping</p>  <p>I can see that 3 groups of 8 equal 24. $3 \times 8 = 24$, $24 \div 8 = 3$</p> <p>By sharing</p>  <p>I can see that 8 groups of 3 equal 24. $8 \times 3 = 24$, $24 \div 8 = 3$</p> <p>Leading to fractional understanding – see Year 2 progression and extend into further fractions.</p>



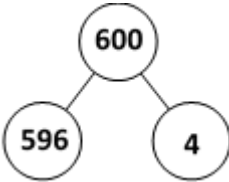
Year 3
Numbers to 1000

Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core concept: COUNTING and PLACE VALUE</p> <p>Core skill: COUNTING ON and BACK</p> <p>Counting on and back in tens and hundreds from any number allowing children to notice in the pattern what changes as a result (and what doesn't change).</p> <div>       </div> <div> <p>236 2 hundreds, 3 tens and 6 ones</p> <p>226 2 hundreds, 2 tens and 6 ones</p> <p>216 2 hundreds, 1 ten and 6 ones</p> <p>206 2 hundreds, 0 tens and 6 ones</p> <p>196 1 hundred, 9 tens and 6 ones</p> <p>186 1 hundred, 8 tens and 6 ones</p> </div>	<p>Core concept: COMPARISON</p> <p>Core skill: COUNTING ON and BACK</p> <p>Count on to find the difference drawing out the use of complements to benchmark numbers e.g. $916 - 897$.</p> <div> <p>916</p> <p>897</p> <p>+ 3 + 16</p> <p>900 916</p> </div> <p>There is 3 more to 900 and 16 more to 916. $3 + 16 = 19$</p>	<p>Core concept: UNITISING and SCALING</p> <p>Core skill: COUNTING ON and BACK</p> <div>  <p>0 8 32</p> </div> <p>Where on the stick will we place 32? Explain how you know.</p> <div>  <p>0 7 14 21 28 35 42 49 56 63 70</p> </div> <p>If I know $\times 1$, $\times 2$, $\times 5$, $\times 10$, what else can I work out?</p> <div> <p>1 more, 1 less</p> <p>2 more, 2 less</p> <p>$\times 4$ $\times 6$ $\times 9$ $\times 3$ $\times 7$ $\times 8$ $\times 4$ $\times 3$</p> </div>

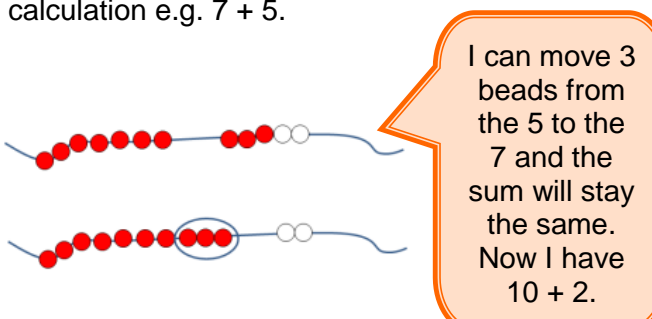
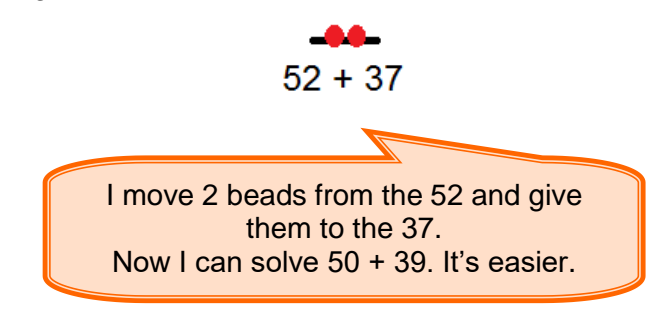
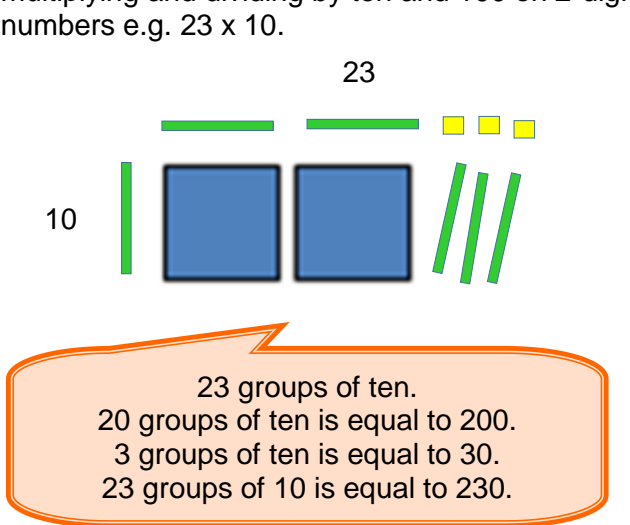
Year 3
Numbers to 1000

Number and Place Value	Addition and Subtraction	Multiplication and Division
<div data-bbox="103 204 763 304"> <p>Core concept: MAGNITUDE and COMPARISON</p> </div> <div data-bbox="103 320 763 528">  </div> <div data-bbox="103 552 763 911"> <p>Number magnitude drawing out the concepts of relative size, order and comparison of number.</p>  <p>I think that number A is 200 because it is nearly 250.</p> </div> <div data-bbox="103 951 763 1230"> <p>Estimation drawing out concepts of distance of numbers to target numbers to prepare for rounding.</p>  <p>200 and</p> <p>234 is 4 from 230 and 6 from 240. 234 is nearer to 230 than to 240.</p> </div>	<div data-bbox="786 204 1447 268"> <p>Core concept: COMPARISON</p> </div> <div data-bbox="786 284 1447 347"> <p>Core skill: COMPENSATION</p> </div> <div data-bbox="786 379 1447 1066"> <p>Part whole model</p> <p>45 - 17</p>  <p>45 - 17</p> <p>= 45 - 20 + 3 = 25 + 3 = 28</p> <p>Subtracting 17 is the same as subtracting 20 and adding 3 back.</p> </div>	<div data-bbox="1469 204 2130 300"> <p>Core concept: COMPARISON and SCALING</p> </div> <div data-bbox="1469 316 2130 379"> <p>Core skill: COMPENSATION</p> </div> <div data-bbox="1469 411 2130 970"> <p>$9 \times 3 = 10 \times 3 - 3$ $9 \times 3 = 10 \times 3 - 1 \times 3$</p>  <p>Nine groups of three are equal to 10 groups of three, less 1 group of three.</p> </div>

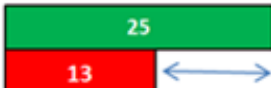

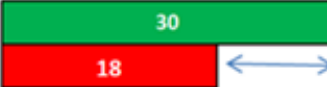
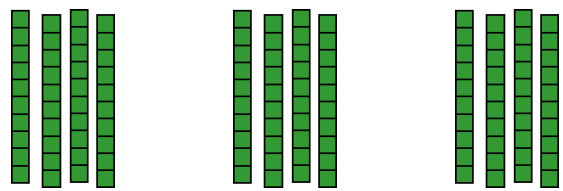
Year 3
Numbers to 1000

Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core skill: ROUNDING</p> <p>Leading to rounding to the nearest 10, the nearest 100 and, later, the nearest 1.</p> <p>359 is nearer to 400 than 300 so it rounds to 400.</p>  <p>315 is nearer to 300 than 400 so it rounds to 300.</p>	<p>Core concept: COMPARISON</p> <p>Core skill: COMPENSATION</p> <p>£3 and 40p add 80p £3 and 40p</p>  <p>£3.40 + £1 - 20p = £4.40 - 20p = £4.20</p> <p>Adding 80p is the same as adding £1 and subtracting 20p.</p> <p>632 - 596</p>  <p>Subtracting 596 is the same as subtracting 600 and adding 4 back.</p>	

Year 3
Numbers to 1000

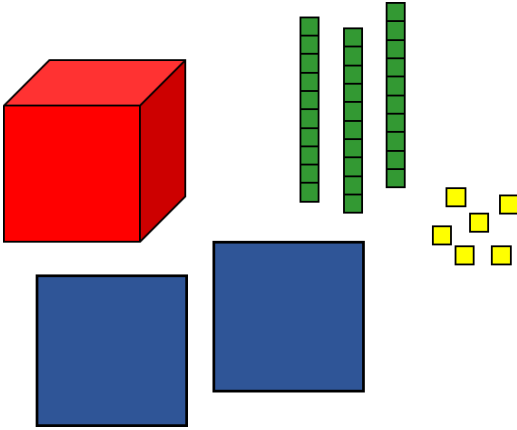
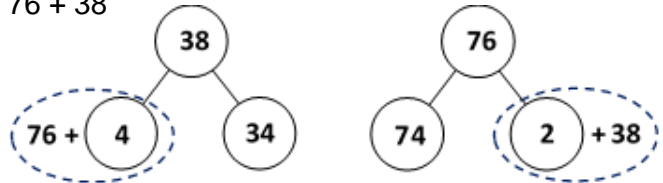

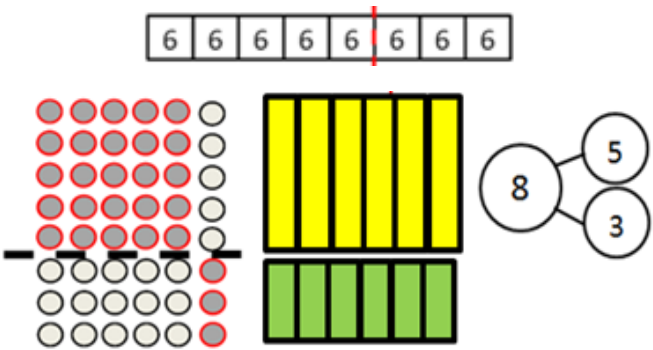
Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Core concept: CONSERVATION and SUM</p> <p>Core skill: REBALANCING</p> <p>Equal sum drawing out the idea of equality when we rebalance the numbers in an addition calculation e.g. $7 + 5$.</p>  <p>e.g. $52 + 37$</p> 	<p>Core concept: PLACE VALUE and SCALING</p> <p>Core skill: $\times \div$ BY 10</p> <p>Check pupils understand the concept of multiplying and dividing by 1 and 0.</p> <p>Place value drawing out the implications of multiplying and dividing by ten and 100 on 2-digit numbers e.g. 23×10.</p> 

Year 3
Numbers to 1000

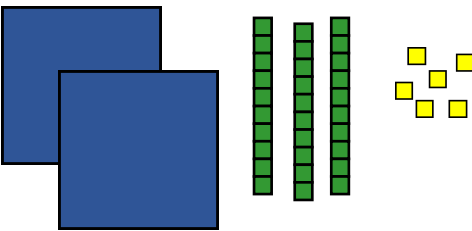
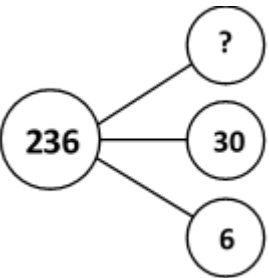
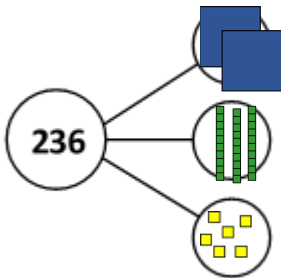
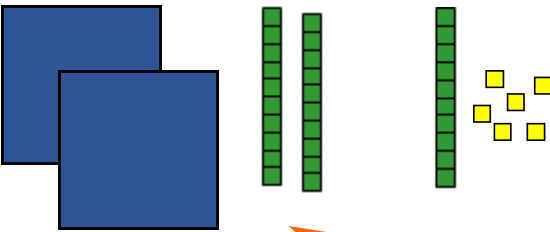
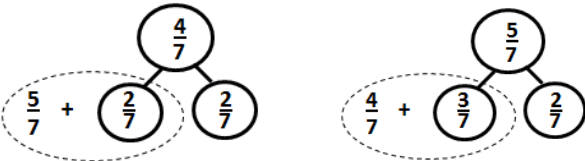
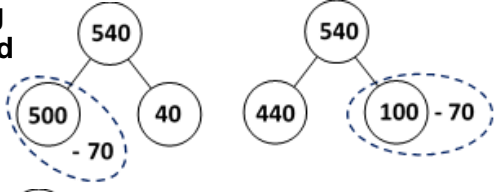
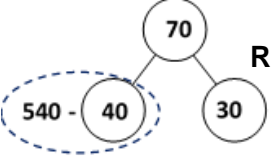
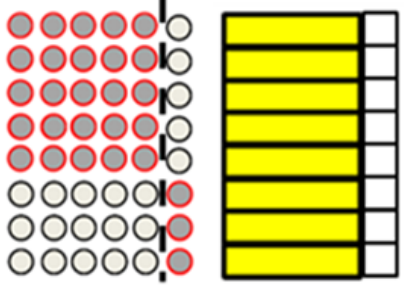
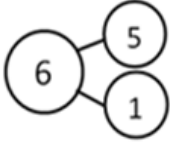
Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Core concept: COMPARISON</p> <p>Core skill: REBALANCING</p> <p>Equal difference using comparison drawing out the concept that adding or subtracting the same quantity from both the subtrahend and minuend maintains the difference between the numbers.</p> <div>    <p>I can take 3 from each number and the difference will remain equal.</p> <p>I can add 5 to each number and the difference will remain equal.</p> <p>If I wanted to solve $21 - 16$, I can take 1 from each number and solve it as $20 - 15$. That is an easier calculation.</p> </div>	<p>$\square = 3 \times 40$</p> <div>  </div> <p>3 groups of 4 tens is equal to 12 tens. 12 tens is 120. So $120 = 3 \times 40$</p> <p>$\square = 120 \div 3$</p> <p>120 is 12 tens. 12 tens divided into 3 groups is equal to 4 tens. 4 tens is 40. So $40 = 120 \div 3$.</p>

Year 3
Numbers to 1000

Year 4

Number and Place Value	Addition and Subtraction	Multiplication and Division
Core concept: UNITISING	Core concept: UNITISING	Core concept: CONSERVATION and SCALING
Core skill: REGROUPING	Core skill: REGROUPING	Core skill: REGROUPING
<p>Grouping thousands, hundreds, tens and ones drawing out the concept that ten ones are equal to a unit of 'one ten' and ten tens are equal to a unit of one hundred etc.</p>  <p>I can see one thousand, two hundreds, three tens and six ones. $1000 + 200 + 30 + 6$ It is thirty-six more than one thousand and two hundred.</p>	<p>Think Regroup for addition</p> <p>Part whole drawing out the concept of regrouping numbers to allow bridging through hundreds, tens and ones. Ask pupils to reason why they may wish to reorder the numbers.</p> <p>Pupils should continue Year 3 learning and be encouraged to explore multiple ways of regrouping both addends (refer to number and place value experiences). Only a limited example is shown here.</p> <p>$76 + 38$</p>  <p>This can be adapted to 'Think 100'</p> <p>$376 + 158$</p> 	<p>Think 5x fact Application of the distributive law</p> <p>Regrouping the multiplier (number of groups). For example, 8×6.</p>  <p>I can use my 5 fact to solve 8×6. I know 5×6 is 30. 3×6 is 18. So 8×6 is 48.</p> <p>$8 \times 6 = 5 \times 6 + 3 \times 6$</p>

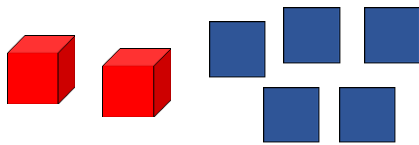
Year 4
Numbers to 10000

Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Part part whole Regroup 3-digit and 4-digit integers flexibly and in multiple ways.</p>     <p>236 can be regrouped into 220 and 16.</p>	<p>Adapt 'Think Regroup' strategy to decimal and fractional part whole as well as measures such as time and money.</p> <p>For example: $\frac{4}{7} + \frac{5}{7} =$</p> <p>Both addends can be regrouped using complements to 1 and 'some more'.</p>  <p>Think Regroup for subtraction</p> <p>Part whole drawing out the skill of regrouping either the minuend or the subtrahend. Pupils should be encouraged to explore multiple ways of regrouping both the minuend and subtrahend (refer to number and place value experiences). For example, 540 - 70.</p> <p>Regrouping the minuend</p>  <p>Regrouping the subtrahend</p> 	<p>Regrouping the multiplicand (number in the group) e.g. 8 x 6.</p>   <p>I can use my 5 fact to solve 8 x 6. I know 8 x 5 is 40. 8 x 1 is 8. So 8 x 6 is 48.</p> <p>$8 \times 6 = 8 \times 5 + 8 \times 1$</p> <p>Regrouping used for multiple strategies.</p> <p>7 x 8</p> <p>I can find 7 x 8 in lots of ways if I didn't know it. I can try 7 x 4 x 2 or 8 x 8 - 8.</p>

Year 4
Numbers to 10000

Number and Place Value

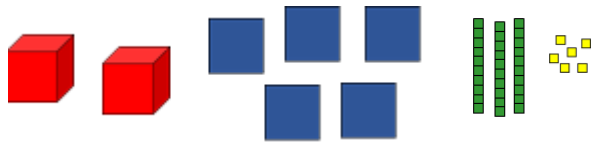
Use resources to prove statements such as:
'There are 25 hundreds in the number 2500'.



Continue this to explore tenths and hundredths through the same concrete and pictorial representations to secure understanding of the multiplicative relationship.



I can see three tenths and four hundredths.



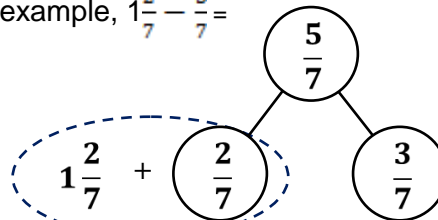
I can see that in the number 25.36, there are 2 tens, 5 ones, 3 tenths and 6 hundredths.

We can also say there are 253 tenths and 6 hundredths in my number.

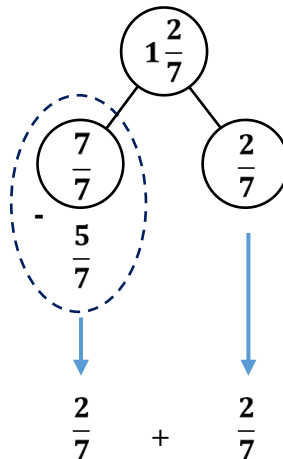
Addition and Subtraction

Then adapted to decimal and fractional part whole as well as measures such as time and money.

For example, $1\frac{2}{7} - \frac{5}{7} =$



I can regroup the subtrahend $\frac{5}{7}$ into $\frac{2}{7}$ and $\frac{3}{7}$.
Then I can take away the $\frac{2}{7}$ leaving $\frac{7}{7}$ or 1
and finally take away $\frac{3}{7}$.



... or I could regroup the minuend, subtract from the 1 and then recombine.

Multiplication and Division

Core skill: APPLYING THE INVERSE

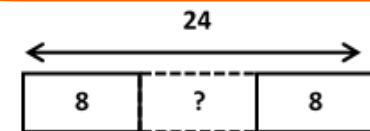
Think multiplication for division

For example, $24 \div 8$.

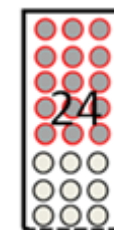
By grouping



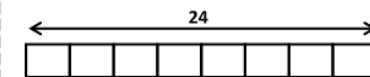
I can see that 3 groups of 8 equal 24.
 $3 \times 8 = 24$, $24 \div 8 = 3$



By sharing






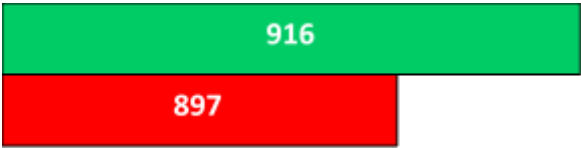







I can see that 8 groups of 3 equal 24.
 $8 \times 3 = 24$, $24 \div 8 = 3$






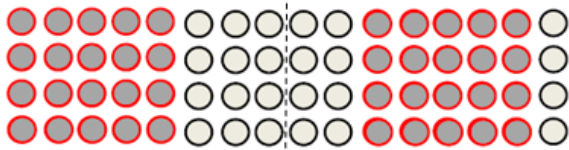

Leading to fractional understanding – see Year 2 progression and extend into further fractions.

Year 4
Numbers to 10000

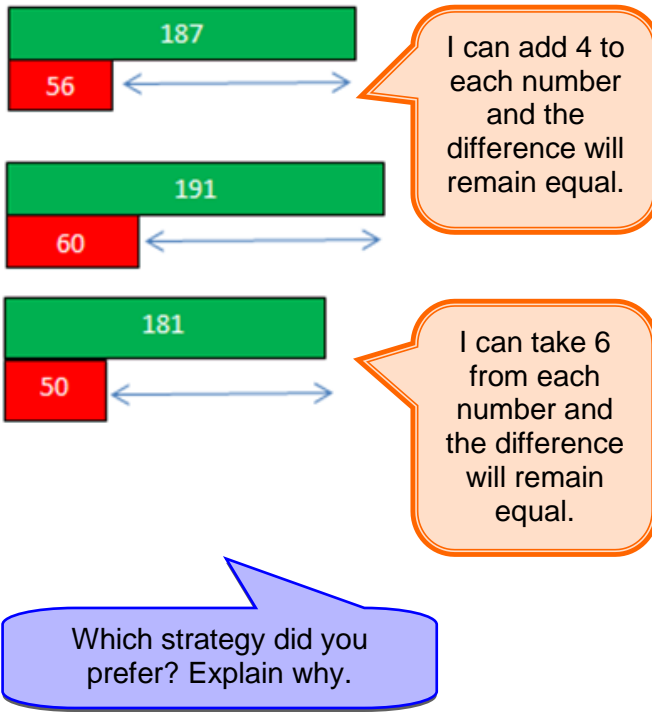
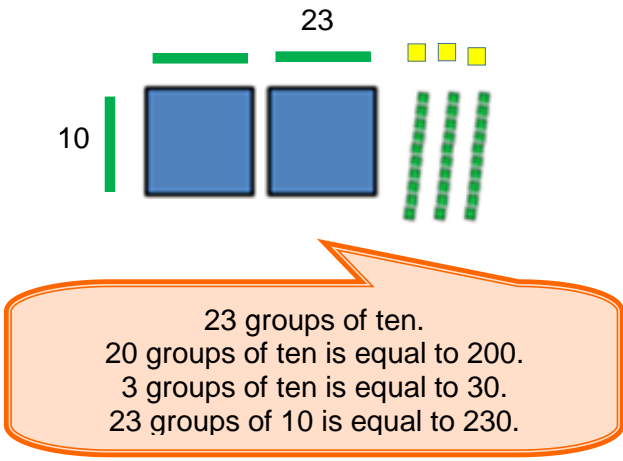
Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core concept: COMPARISON</p> <p>Core skill: COUNTING ON and BACK</p> <p>Counting on and back in hundreds and tens from any number. Allow pupils to notice the pattern. What changes as a result and what does not.</p> <div>  2236 </div> <div>  2136 </div> <div>  2036 </div> <div>  1936 </div> <div>  1836 </div> <p>Show me the difference between 2036 and 2136.</p> <p>Counting on and back in multiples as well as making counting links e.g. counting in 6s, 60s (relate to time), 600s and 0.6s.</p> <p>Counting in 25s, 50s, 0.1s and 0.01s paying attention to bridging (regrouping) points.</p>	<p>Core concept: COMMUTATIVITY and UNITISING</p> <p>Core skill: REORDERING and FINDING COMPLEMENTS</p> <p>Adding three or more numbers. Draw out the reasons why pupils may wish to reorder the numbers and focus on a range of strategies used.</p> $800 + 240 + 360 =$ $2.5 + 25 + 5 + 2.5 =$ $310 + 700 + 300 =$ <p>Core concept: COMPARISON and DIFFERENCE</p> <p>Core skill: COUNTING ON and BACK</p> <p>916 – 897</p> <p>Comparison Drawing out use of benchmark numbers.</p> <div>  </div>	<p>Core concept: UNITISING and SCALING</p> <p>Core skill: COUNTING ON and BACK</p> <div>  <p>Where on the counting stick will we place 32? Explain your thinking.</p> </div> <div>  </div> <p>If I know x1, x2, x5, x10, what else can I work out?</p> <div> <div> <p>1 more, 1 less</p> <div> <p>x4 x6</p> <p>x9 x3</p> </div> </div> <div> <p>2 more, 2 less</p> <div> <p>x7 x8</p> <p>x4 x3</p> </div> </div> </div> <p>Year 4 Numbers to 10000</p>

Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core concept: MAGNITUDE</p> <p>Number magnitude drawing out the concepts of relative size, order and comparison of number.</p>  <p>The number couldn't be...because It could be ... because...</p>  <p>0.5</p> <p>Number estimation (using scales) should be applied to different scales of measures. This should include those with negative and dial scales.</p> <p>Core skill: ROUNDING</p> <p>Leading to rounding to the nearest 10, 100, 1000, hour and £1 etc.</p>	<p>Core concept: COMPARISON, SUM and DIFFERENCE</p> <p>Core skill: COMPENSATION and REBALANCE</p> <p>Equal sum drawing out the concept of equality when rebalancing the numbers in an addition calculation.</p> <p>255 + 49 is easier if I take one from the 255 and give it to the 49. My sum stays equal. Then my sum becomes 254 + 50 = 304.</p> <p>Compensation with the same calculation supports pupils' multi-strategy approach. Pupils can begin to evaluate strategies.</p> <p>I could also think of 255 + 49 as compensation because adding 49 is like adding 50 and taking one away. Now my sum looks like this: 255 + 50 - 1 = 304.</p>	<p>Core concept: COMPARISON and SCALING</p> <p>Core skill: COMPENSATION</p> <p>$3 \times 9 = 3 \times 10 - 3$</p>  <p>Nine groups of three is equal to ten groups of three, less 1 group of 3.</p> <p>I could use this to find 90×3. $100 \times 3 - 10 \times 3$</p>

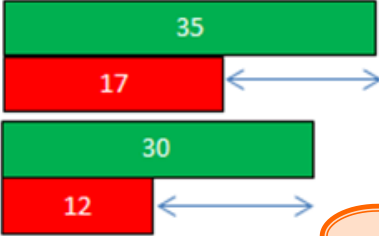
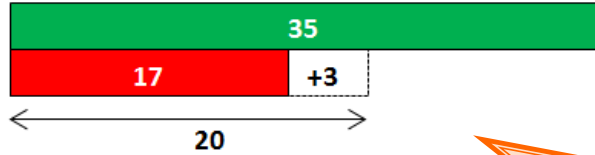
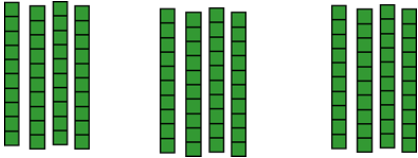
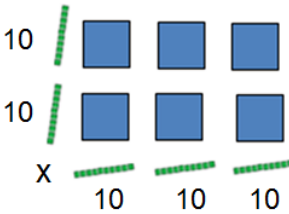
Year 4
Numbers to 10000

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Rebalancing</p>  <p>It is 1.44pm. What time will it be in 50 minutes?</p> <p>10 minutes</p>  <p>1 hour 44 minutes + 50 minutes = 1 hour 34 minutes + 60 minutes = 2 hours and 34 minutes = 2.34pm</p> <p>If I give 10 minutes from the 1 hour 44 to the 50 minutes then I can add on 1 hour.</p> <p>Compensation</p>  <p>It is 1.44pm. What time will it be in 50 minutes?</p> <p>Adding 50 minutes is like adding one hour and taking away 10 minutes.</p>	<p>Core concept: CONSERVATION and SCALING</p> <p>Core skill: REARRANGING</p> <p>Children should explore rearranging arrays to simplify multiplications but conserve the area.</p> <p>$4 \times 16 =$</p>  <p>Can become 8×8</p>  <p>Doubling the 4x and halving the group of 16 gives me 8×8. The area remains equal.</p>

Year 4
Numbers to 10000

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Equal difference using comparison drawing out the concept that adding or subtracting the same quantity from both the subtrahend and minuend maintains the difference between the numbers.</p> <div data-bbox="795 359 1444 1077">  </div>	<div data-bbox="1473 167 2134 236">Core concept: PLACE VALUE</div> <div data-bbox="1473 252 2134 320">Core skill: $x \div \text{BY } 10, 100$</div> <p>Check pupils understand the concept of multiplying and dividing by 1 and 0.</p> <p>Place value drawing out the implications of multiplying and dividing by 10 and 100 on 2-digit numbers e.g. 23×10.</p> <div data-bbox="1496 566 2116 1029">  </div>

Year 4
Numbers to 10000

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>I had £35. I bought a book for £17. How much change do I get?</p>  <p>I can take 5 from each number and the difference will remain equal.</p> <p>Compare this to compensation.</p>  <p>Subtracting £17.00 is like subtracting £20.00 and adding back £3.00.</p>	<p>$\square = 3 \times 40$</p>  <p>3 groups of 4 tens is equal to 12 tens. 12 tens is 120. So $120 = 3 \times 40$.</p> <p>$\square = 120 \div 3$</p> <p>120 is 12 tens. 12 tens divided into 3 groups is equal to 4 tens. 4 tens is 40. So $40 = 120 \div 3$.</p> <p>$20 \times 30 = \square$</p>  <p>When I multiply 10 by 10 it equals 100. 2 tens multiplied by 3 tens is equal to 6 hundreds.</p>

Year 4
Numbers to 10000

Lower KS2 examples

<p>'Think Regroup' for addition</p> <p>Think 10</p> $\begin{array}{r} 37 + 45 \\ 76 + 27 \\ 537 + 8 \end{array} \quad \begin{array}{r} 68 + 23 \\ 55 + 16 \\ 727 + 5 \end{array} \quad \begin{array}{r} 29 + 75 \\ 42 + 38 \\ 213 + 18 \end{array} \quad \begin{array}{r} 146 + 37 \\ \square 7 - 45 = 32 \end{array}$ <p>$36 - \square = 29$ $56 - 2\square = 33$</p> <p>Think 100</p> $\begin{array}{r} 290 + 13 \\ 270 + 51 \\ 86 + 770 \end{array} \quad \begin{array}{r} 370 + 50 \\ 67 + 350 \\ 680 + 63 \end{array} \quad \begin{array}{r} 580 + 73 \\ 860 + 69 \end{array}$ <p>Think 1000</p> $\begin{array}{r} 4900 + 500 \\ 3200 + 910 \\ 3700 + 370 \end{array} \quad \begin{array}{r} 4800 + 260 \\ 230 + 7900 \\ 3622 + 500 \end{array} \quad \begin{array}{r} 6900 + 430 \\ 570 + 8500 \end{array}$ <p>Think 1</p> $2.7 + 1.4 \quad 2\frac{8}{10} + \frac{3}{10} = \quad 6.5 + 5.6$ $1\frac{7}{8} + 1\frac{5}{8}$	<p>Re-ordering and finding complements</p> <p>Complements to 10</p> $\begin{array}{r} 8 + 6 + 2 + 3 + 4 \\ 1 + 4 + 6 + 7 + 9 \\ 25 + 50 + 5 \end{array} \quad \begin{array}{r} 3 + 5 + 7 + 5 + 4 \\ 30 + 50 + 70 \\ 75 + 40 + 20 + 25 \end{array}$ <p>Complements to 100</p> $\begin{array}{r} 400 + 547 + 600 \\ 750 + 400 + 250 \end{array} \quad \begin{array}{r} 700 + 240 + 300 \end{array}$ <p>Complements to 1</p> $2.7 + 4 + 1.3 \quad 4.6 + 5 + 2.4 \quad 8.2 + 3 + 5.8$ <p>Compensation</p> $\begin{array}{r} 42 + 29 \\ 232 + 49 \\ 132 + 59 \\ 405 + 199 \\ 43 - 18 \\ 483 - 99 \\ 401 - 97 \\ 298 - 156 \end{array} \quad \begin{array}{r} 45 + 27 \\ 856 + 17 \\ 568 + 195 \\ 597 + 308 \\ 94 - 37 \\ 256 - 98 \\ 736 - 301 \\ 799 - 403 \end{array} \quad \begin{array}{r} 24 + 47 \\ 48 + 325 \\ 399 + 423 \\ 54 - 29 \\ 398 - 74 \\ 613 - 299 \end{array} \quad \begin{array}{r} 28 + 65 \\ 232 + 95 \\ 412 + 298 \\ 77 - 9 \\ 597 - 63 \\ 743 - 397 \end{array} \quad \begin{array}{r} 68 + 27 \\ 82 - 23 \end{array}$ <p>Rebalancing - Equal sum</p> $\begin{array}{r} 45 + 27 \\ 73 + 39 \\ 368 + 123 \\ 3.7 + 1.9 \end{array} \quad \begin{array}{r} 26 + 39 \\ 84 + 47 \\ 404 + 198 \\ 7.6 + 4.7 \end{array} \quad \begin{array}{r} 78 + 18 \\ 42 + 97 \\ 356 + 427 \\ 1.9 + 5.8 \end{array} \quad \begin{array}{r} 65 + 27 \\ 116 + 35 \\ 528 + 298 \end{array}$ <p>Rebalancing - Equal difference</p> $\begin{array}{r} 75 - 28 \\ 83 - 21 \\ 912 - 797 \\ 482 - 302 \end{array} \quad \begin{array}{r} 56 - 29 \\ 75 - 12 \\ 837 - 498 \\ 729 - 404 \end{array} \quad \begin{array}{r} 78 - 38 \\ 95 - 42 \\ 711 - 467 \\ 548 - 202 \end{array} \quad \begin{array}{r} 55 - 27 \\ 67 - 51 \\ 628 - 198 \\ 637 - 203 \end{array}$ <p>$6.4 - 3.9 \quad 6.6 - 3.2 \quad 7.7 - 4.8 \quad 1\frac{2}{7} - \frac{5}{7}$</p> <p>Counting on to subtract</p> $\begin{array}{r} 315 - 298 \\ 412 - 396 \\ 917 - 898 \end{array} \quad \begin{array}{r} 611 - 598 \end{array}$	<p>Think multiplication</p> $\begin{array}{r} 85 \div 5 \\ 660 \div 3 \\ \square \div 3 = 8 \end{array} \quad \begin{array}{r} 72 \div 4 \\ 210 \div 7 \\ 3\square \div 5 = 6 \end{array} \quad \begin{array}{r} 99 \div 6 \\ 540 \div 9 \end{array} \quad \begin{array}{r} 240 \div 12 \\ 500 \div 4 \end{array}$ <p>X and \div 10, 100 and 1000</p> $\begin{array}{r} 4 \times 30 \\ 300 \times 4 \\ 3 \times 2000 \\ 500 \div 10 \\ 120 \div \square = 12 \end{array} \quad \begin{array}{r} 9 \times 30 \\ 800 \times 7 \\ 4000 \times 6 \\ 400 \div 5 \\ 365\text{cm} = \square \text{ m} \end{array} \quad \begin{array}{r} 70 \times 70 \\ 9 \times 800 \\ 8 \times 7000 \\ 600 \div 2 \\ 750\text{mm} = \square \text{ cm} \end{array} \quad \begin{array}{r} 60 \times 50 \\ 6 \times 400 \\ 9 \times 8000 \\ 240 \div 4 \end{array}$ <p>Double and near double facts</p> $\begin{array}{r} 7 \times 20 \\ 16 \times 20 \\ 80 \div 4 \end{array} \quad \begin{array}{r} 3 \times 38 \\ 18 \times 2000 \\ 160 \div 4 \end{array} \quad \begin{array}{r} 9 \times 200 \\ 1600 \div 4 \end{array} \quad \begin{array}{r} 11 \times 4 \\ 2400 \div 4 \end{array}$ <p>Think 5 / Think 10 for multiplication</p> $\begin{array}{r} 28 \times 5 \\ 13 \times 21 \\ 11 \times 314 \end{array} \quad \begin{array}{r} 16 \times 8 \\ 34 \times 19 \\ 21 \times 400 \end{array} \quad \begin{array}{r} 23 \times 9 \\ 123 \times 4 \\ 400 \times 38 \end{array} \quad \begin{array}{r} 92 \times 8 \\ 214 \times 6 \end{array} \quad \begin{array}{r} 52 \times 4 \\ 9 \times 234 \end{array}$
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Key facts

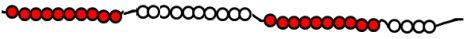
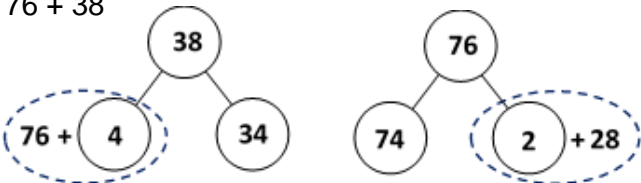

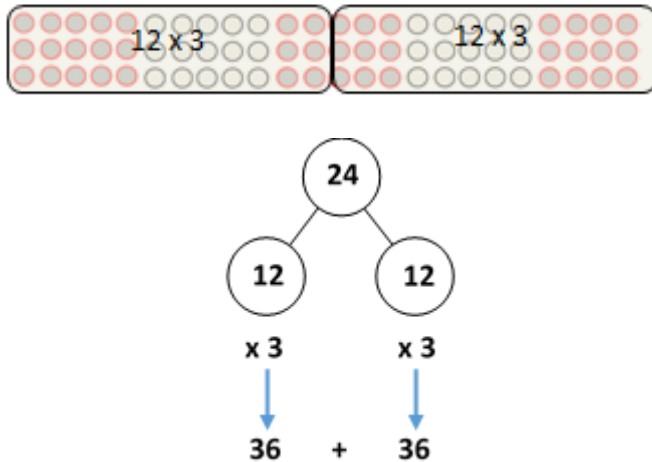
Year Three Recall

- Sums and differences between pairs of numbers which are multiples of 10 and 100
- Doubles and halves of multiples of 10 or 100
- Complements to 100
- Complements to 60 (time)
- Complements of tenths that make 1
- Complements of fractions with the same denominator that make 1 e.g. $\frac{3}{7} + \frac{4}{7} = 1$
- $\times 3$, $\times 4$, $\times 8$ facts including division facts
- Number of seconds in a minute
- Number of days in a month and in a year including a leap year



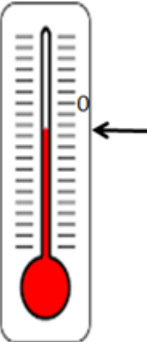


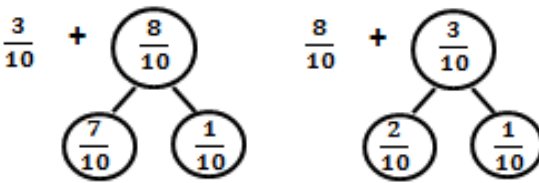
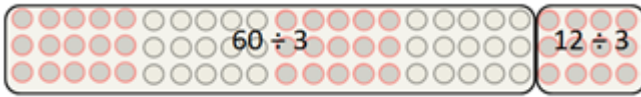
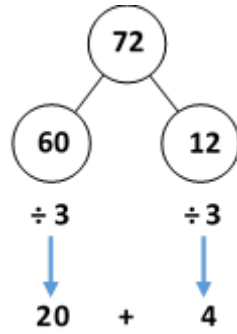
Year Four Recall

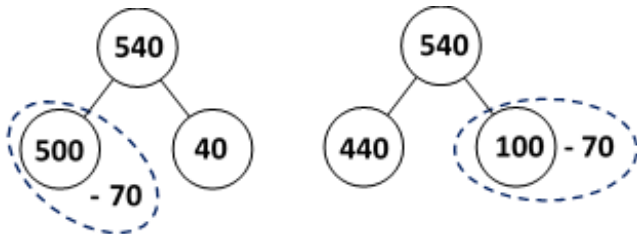
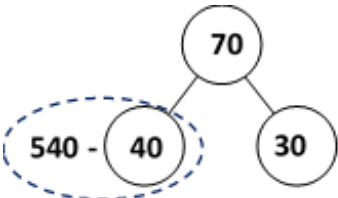
- Review addition and subtraction facts within 20, ensure application to 10, 100 and 1000 ($6 + 3$, $60 + 30$, $600 + 300$, $6000 + 3000$)
- Doubles and halves of multiples of 10, 100 or 1000 ($6 + 6$, $60 + 60$, $600 + 600$, $6000 + 6000$)
- All multiplication and division facts to 12×12
- Multiplication and division by zero and one facts
- Division and multiplication by 10 and 100
- Conversion of kilometres to metres, hours to minutes, years to months, weeks to days
- Complements of hundredths that make 1

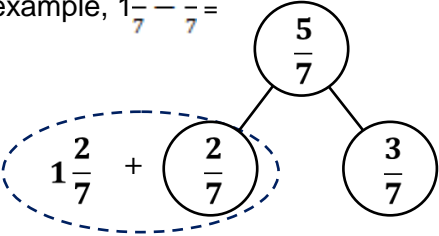
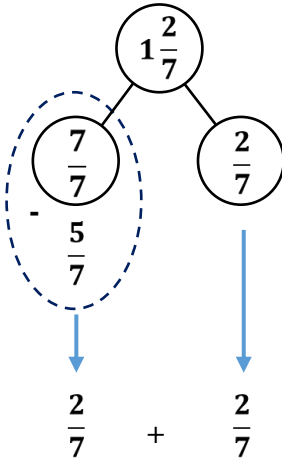
Years 5 and 6

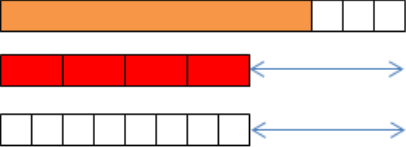
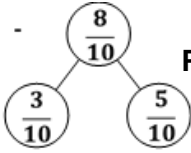
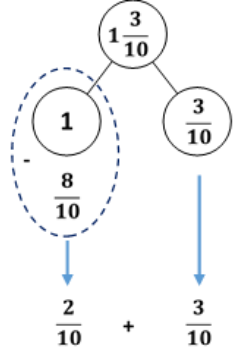
Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Core concept: MAGNITUDE and COMPARISON</p> <p>Introduce pupils to a range of calculations in which a secure understanding of place value is required to support the solution.</p> <p>For example, $2.005 + 3.24$.</p> <p>Estimation drawing out the concept of 'distance' of numbers to target numbers / benchmarks in preparation for rounding.</p> <p>20,000 and </p> <p>I can see that 20,034 is 4 from 20,030 and 6 from 20,040. 20,034 is nearer to 20,030 than to 20,040.</p> <p>Comparison to benchmark numbers Using number knowledge to look for 'nearly numbers' in calculations.</p> <p>$7834 + 79,996$</p> <p>79,996 is 4 less than 80,000 and that's an easier number to add.</p>	<p>Core concept: UNITISING</p> <p>Core skill: REGROUPING</p> <p>Think Regroup for addition Part whole drawing out the skill of regrouping numbers to allow bridging through hundreds, tens and ones. Ask pupils to reason why they may wish to reorder the numbers.</p> <p>Pupils should continue LKS2 learning and be encouraged to explore multiple ways of regrouping both addends (refer to number and place value experiences). Only a limited example is shown here.</p> <p>$76 + 38$</p>  <p>$376 + 158$</p> 	<p>Core concept: UNITISING</p> <p>Core skill: REGROUPING</p> <p>Think Regroup for multiplication and division drawing out the distributive law for both multiplication and division and encouraging pupils to regroup and multiply in a variety of ways, evaluating the most useful.</p> <p>$24 \times 3 =$</p> 


Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Rounding</p> <p>Round 136,521 to the nearest 100, 1000 and 10,000.</p> <p>Rounding as estimation for multiplication and division.</p> <p>688 x 79 =</p> <div data-bbox="107 603 759 738" style="border: 1px solid orange; border-radius: 15px; padding: 10px; margin: 10px 0;"> <p>688 rounds to 700 and 79 rounds to 80. The calculation 688 x 79 is close to 700 x 80, which is 56,000.</p> </div> <p>789 ÷ 79 =</p> <div data-bbox="107 1010 759 1145" style="border: 1px solid orange; border-radius: 15px; padding: 10px; margin: 10px 0;"> <p>789 rounds to 800 and 79 rounds to 80. The calculation 789 ÷ 80 is close to 800 ÷ 80, which equals 10.</p> </div>	<p>Then adapted to decimal and fractional part whole as well as measures such as time and money.</p> <p>For example, $\frac{4}{7} + \frac{5}{7} =$</p> <p>Here both addends can be regrouped using complements to 1 and some more.</p> <div data-bbox="801 507 1415 699" style="text-align: center;"> </div> <p>Extend into UKS2 by converting fractions into equivalents with common denominators. Beginning with conversions where no regrouping is required.</p> <p>For example:</p> <div data-bbox="862 1061 1373 1217" style="text-align: center;"> $\frac{2}{10} + \frac{2}{5} =$ $\frac{2}{10} + \frac{4}{10} =$ </div>	<p>24 x 3 =</p> <div data-bbox="1480 263 2123 355" style="text-align: center;"> </div> <div data-bbox="1686 384 1919 715" style="text-align: center;"> </div>

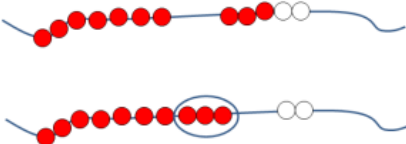
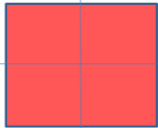
Number and Place Value	Addition and Subtraction	Multiplication and Division
<p>Number magnitude drawing out the concepts of relative size, order and comparison of number.</p>  <p style="text-align: center;">0.5</p> <p>Number estimation using scales should continue to be applied to scales of measurement including those with negative and dial scales.</p>  <p>10:35</p> <p>← ?</p> <p>½ litre</p>  <p>0</p>	<p>Progress to examples where regrouping would be a valid strategy.</p> $\frac{3}{10} + \frac{4}{5} =$  $\frac{3}{10} + \frac{8}{10} =$  <p>Rehearse regrouping either addend to make 1s and some more.</p> 	<p>$72 \div 3 =$</p>  <p>$60 \div 3$ $12 \div 3$</p>  <p>$15 \times 3.4 =$</p> <div style="border: 2px solid orange; border-radius: 15px; padding: 10px; margin-top: 20px;"> <p>I know that $10 \times 3.4 = 34$ Then I can halve 34 to find 5 groups of 3.4 which is 17. After that, I have to recombine the products. This equals 51.</p> </div>

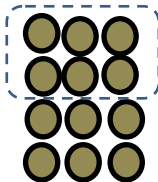
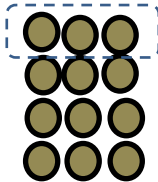
Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Think regroup for subtraction</p> <p>Part whole drawing out the skill of regrouping either the minuend or the subtrahend.</p> <p>Pupils should be encouraged to explore multiple ways of regrouping both the minuend and subtrahend (refer to number and place value experiences).</p> <p>For example, $540 - 70$</p> <p>Regrouping the minuend</p>  <p>Regrouping the subtrahend</p> 	

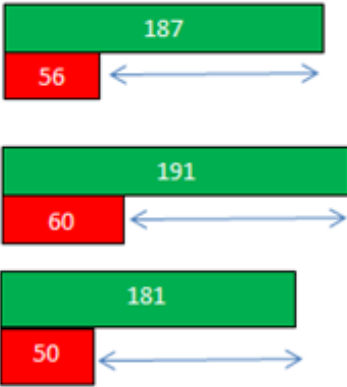
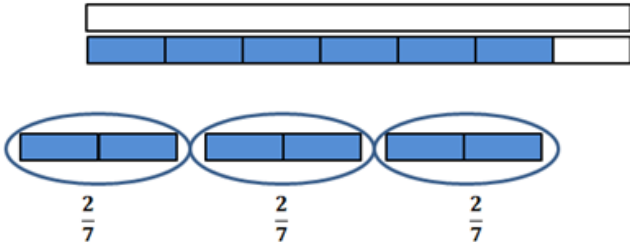
Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Then adapted to decimal and fractional part whole as well as measures such as time and money. For example, $1\frac{2}{7} - \frac{5}{7} =$</p>  <p>I can regroup the subtrahend $\frac{5}{7}$ into $\frac{2}{7}$ and $\frac{3}{7}$. Then I can take away the $\frac{2}{7}$ leaving $\frac{7}{7}$ or 1 and finally take away $\frac{3}{7}$.</p>  <p>... or I could regroup the minuend, subtract from the 1 and then recombine.</p>	

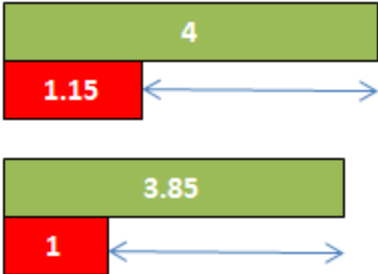
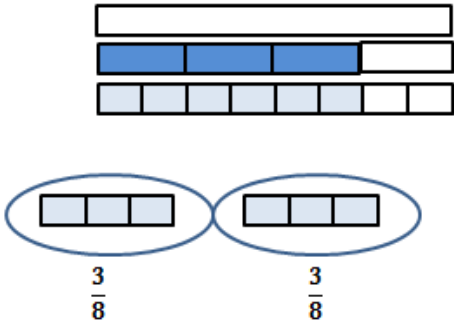
Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Extend into UKS2 by converting fractions into equivalents with common denominators.</p> <p>Beginning with conversions where no regrouping is required. For example, $\frac{2}{10} - \frac{1}{20} =$</p> <p>Progress to examples where regrouping would be a valid strategy. For example, $1\frac{3}{10} - \frac{4}{5} =$</p> <p>Pupils will have to know that $\frac{4}{5} = \frac{8}{10}$ before they can solve the calculation.</p> <p>Then they could regroup either the subtrahend or the minuend.</p>  <p>For example, $1\frac{3}{10} - \frac{8}{10} =$</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> $1\frac{3}{10} - \frac{8}{10}$  <p>Partitioning the subtrahend</p> </div> <div style="text-align: center;">  <p>Partitioning the minuend</p> </div> </div>	

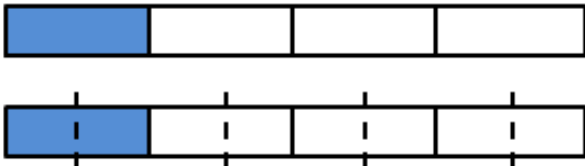
Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Core concept: CONSERVATION</p> <p>Reordering and finding complements across a range of numbers.</p> <p>For example:</p> $\begin{array}{r} 47 + 603 \\ 0.15 + 1.85 \end{array} \quad \begin{array}{r} 0.45 + 1.63 \\ \text{£}3.99 + \text{£}7.80 + \text{£}2.01 \end{array}$ <p>Two decimal numbers add together to make a total of 1. One number is 0.0006. What is the other number?</p>	<p>Core concept: CONSERVATION</p> <p>Core skill: REARRANGING</p> <p>Factorisation drawing on the associative law for multiplication and related division facts.</p> <p>For example, $24 \times 3 = 12 \times 3 \times 2$</p>  <p>Two and twelve are factors of 24 and I find it easier to calculate 12×3 first and then double it.</p> <p>Doubling and halving</p> <p>$12 \times 2.5 =$ $12 \times 2.5 = 6 \times 5$. I halved the 12 and doubled the 2.5 to make the calculation easier.</p> <p>$16 \times 6 \frac{1}{4} =$</p> <p>$16 \times 6 \frac{1}{4} = 8 \times 12 \frac{1}{2} = 4 \times 25 = 100$ I can make this easier for me by doubling and doubling again the $6 \frac{1}{4}$. This means I have to halve and halve again the 16 to maintain the area. Now I get $4 \times 25 = 100$.</p> <p><i>Application to KS2 example (Q11 paper 1 2016):</i> $71 \times 8 = 142 \times 4$ $= 284 \times 2$</p>

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Core concept: CONSERVATION and COMPARISON</p> <p>Core skill: REBALANCING</p> <p>Equal sum drawing out the concept of equality when rebalancing the numbers in an addition calculation.</p>  <p>Pupils use bead strings to demonstrate that: $7 + 5 = 10 + 2$</p> <p>Apply concept to range of numbers and missing number problems.</p> <p>For example, $24 + \square = 30 + 3$.</p> <p>See Year 3 and 4 examples These should include rehearsal using calculations such as:</p> <div> $39 + 52$ $345 + 198$ </div> <div> $0.39 + 6.54$ $5.1 + 2.7 = \square + 4.8$ </div>	<p>Halving and halving for division</p> <p>Once pupils are confident with the 'halve and double' strategy for multiplication, they will try to apply it to division and will need to understand why their answers do not make sense. Stress again the importance of estimation.</p> <p>Investigate the principle of halving and halving with pupils.</p> <div>  $72 \div 4 = (72 \div 2) \div 2$ </div> <div> <p>When I am dividing by 4, I like to halve the number and halve it again.</p> </div> <p>This strategy is best explored through practical contexts so pupils can clearly see that even though the dividend and the divisor are changing the quotient remains constant.</p>

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Ensure pupils are secure with the concept of equal sum before considering questions such as:</p> $7834 + 79,996$ <div> <p>79,996 is 4 away from 80,000. I can rebalance the sum by taking 4 from 7834 and giving it to the 79,996. Now I have $80,000 + 7,830 = 87,830$.</p> </div> <p>Compensation with the same calculation supports pupil's multi-strategy approach. Pupils can continue to evaluate strategies.</p> $7834 + 79,996$ <div> <p>Adding 79,996 is like adding 80,000 and subtracting 4. I can do $80,000 + 7834 - 4 = 87,830$</p> </div> <p>Improve multi-strategy approaches by asking for two different ways of solving calculations such as:</p> $\square = 5,756 + 8,643 \quad 16.98 + 23.214 = \square$	<p>For example, If I shared 12 cookies among 4 children each child would get 3 cookies.</p> $12 \div 4 = 3$ <div>  <p>I can also see that 6 cookies shared between 2 people would give the same group size. The size of the group hasn't changed. So $12 \div 4$ can be changed into $6 \div 2$.</p> </div> <div>  <p>As I am trying to find out the group size, I can also see that $3 \div 1$ gives me the group size. So $12 \div 4$ can be thought of as $6 \div 3$ and $3 \div 1$. I can see all of these in the array.</p> </div> <p>Applying this conceptual understanding to larger numbers encourages playfulness with division.</p> $364 \div 16 =$ $182 \div 8 =$ $91 \div 4 =$ $45.5 \div 2 = 22.75$ <div> <p>I saw that I could halve both the dividend and the divisor, so I did to see if it made it easier. Then I realised that I could halve them again and again.</p> </div> <p><i>Apply core concepts in the context of fractions.</i></p>

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Equal difference using comparison drawing out the concept that adding or subtracting the same quantity from both the subtrahend and minuend will maintain the difference between the numbers.</p> <div>  <div> <p>I can add 4 or take 6 away from each of the numbers and the difference will be the same.</p> </div> </div> <p>Remember to rehearse simple calculations such as $367 - 9$ before applying to a range of numbers.</p> <p>$132,457 - 11,999 =$</p> <div> <p>11,999 is nearly 12,000. If I add one to each number the difference will stay equal. Now my calculation is $132,458 - 12,000 =$</p> </div>	<p>Core concept: UNITISING</p> <p>Core skill: REGROUPING</p> <p>Division of fractions by integers drawing out the concept of multiple groups of the numerator before teaching a rule. Ensure that pupils always refer to the whole.</p> <div> <p>I know that $12 \div 3$ can be thought of as 'If I share 12 equally between 3 groups, how many in each group?'</p> <p>So $\frac{6}{7} \div 3$ can be thought of as, 'If I share $\frac{6}{7}$ equally between 3 groups, how many in each group?'</p> </div> <p>$\frac{6}{7} \div 3$</p> <div>  </div>

Number and Place Value	Addition and Subtraction	Multiplication and Division
	<p>Use a range of examples.</p> <p>$\square = 4 - 1.15$</p>  <p>It is easier if I subtract 0.15 from each number. The difference will stay the same. Now my calculation is $3.85 - 1 =$</p> <p>Compare this to compensation</p> <p>$132,457 - 11,999 =$</p> <p>Subtracting 11,999 is like subtracting 12,000 and then adding 1. Now my calculation is $132,457 - 12,000 + 1 =$</p>	<p>Progress to dividing fractions in which the fraction needs converting.</p> <p>$\frac{3}{4} \div 2$</p>  <p>$\frac{3}{4} \div 2$ can be understood as: "If I share $\frac{3}{4}$ equally between 2 groups, how many in each group?"</p>

Number and Place Value	Addition and Subtraction	Multiplication and Division
		<p>Multiplication of fractions by fractions</p> <p>Equal groups</p> <p>I know that 3×4 could mean 3 groups of 4. So $\frac{1}{2} \times \frac{1}{4}$ means half a group of $\frac{1}{4}$.</p> <p>$\frac{1}{2} \times \frac{1}{4} =$</p>  <p>When we find half of any number, we divide it by two.</p> <p>The blue part has a value of $\frac{1}{4}$. When I halve it, it makes $\frac{1}{8}$.</p> <p>Pupils should focus upon the denominators and reason why, when multiplied, we find the product of the denominators. Once understood pupils can employ the rule.</p>

Number and Place Value	Addition and Subtraction	Multiplication and Division
		<p>Halve and double</p> <p>The 'halve and double' rule can be applied to fractions.</p> <p>Pupils have already secured conceptual understanding of this rule, for example:</p> $5 \times 4 = 10 \times 2 = 20 \times 1$ <p>Apply this understanding to fractions, for example:</p> $\frac{1}{2} \times \frac{1}{4} =$ <p>If we double the first term and halve the second, we can transform the calculation to:</p> $1 \times \frac{1}{8} = \frac{1}{8}$ <p><i>For further detail regarding the multiplication and division of fractions refer to the 'HfL Bar Modelling Progression' document.</i></p>

Upper KS2 examples

<p>Place Value</p> <p>937 + 100 1969 + 100 546 - 40</p> <p>1.7 + 0.05 40 000 - 500</p> <p>246 ÷ 1 100 x 217 0.4 ÷ 10</p> <p>1.68 x 100 100 x 100</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>435 - 30 979 + 100 3.005 + 6.12 2.15 + 0.05</p> <p>100 x 412 0.9 ÷ 10 1.28 x 100 50,000 - 500</p> <p>10 x 100</p> <p>Two decimal numbers add together to equal 1 One of the numbers is 0.007. What is the other number?</p> <p>Circle two numbers that added together make 0.25</p> <p>0.05 0.23 0.2 0.5</p> <p>Circle two numbers that multiply together to equal 1 million</p> <p>200 2,000 5,000 50,000</p> <p>Write the number that is 5 less than 10 million</p> <p>Write the number that is one hundred thousand less than six million</p> <p>Round 124,531 to the nearest 10,000, 1,000, 100</p> <p>Think Regroup</p> <p>58 + 6 5 + 47 630 + 73 680 + 78</p> <p>560 + 89 8900 + 230</p> <p>74 - 7 97 - 8 320 - 50 2300 - 600</p> <p>3400 - 1700</p> <p>5 - 2.65 8.1 - 2.75 $1\frac{2}{5} + \frac{3}{10} =$ $1\frac{3}{10} - \frac{2}{5} =$</p> <p>£3367.40 - £1021.23</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>4 - 1.15 $1\frac{4}{5} + \frac{3}{10}$ $1\frac{1}{4} + \frac{1}{3}$ $1\frac{1}{5} - \frac{1}{4}$ $\frac{3}{4} + \frac{7}{8} =$</p> <p>5,756 + 8,643 936 + 285</p>	<p>Compensation</p> <p>56 + 8 72 + 9 56 - 8 72 - 9</p> <p>371 + 18 255 + 49 304 + 299</p> <p>673 - 99 854 - 398 3720 - 996</p> <p>0.71 + 0.09 0.56 + 0.08 0.34 - 0.09</p> <p>£1.17 + £0.39 £8.89 - £4.99</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>468 - 9 472 - 9 15.98 + 26.314</p> <p>12 - 6.01 15.4 - 8.88</p> <p>Rebalancing - Equal sum</p> <p>56 + 8 72 + 9 371 + 18 255 + 49</p> <p>304 + 267</p> <p>£37.67 + £3.85 563 + 397 890,488 + 4,890</p> <p>229,899 + 31,321</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>89,994 + 7,643 936 + 285 89,994 + 7,643</p> <p>Rebalancing - Equal difference</p> <p>85 - 18 42 - 17 88 - 43 437 - 103</p> <p>819 - 504 532,525 - 9897</p> <p>£122.56 - £87.99 9.1 - 6.7 15.3 - 5.7</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>468 - 9 472 - 9 122,456 - 11,999</p> <p>4 - 1.15 12 - 6.01</p> <p>15.4 - 8.88 234,897 - 45,996</p>	<p>Think Partition for x and ÷</p> <p>32 x 4 29 x 2 122 x 4 4.6 x 2</p> <p>75 x 3 8.3 x 6 39 x 7</p> <p>3.3 x 7 5 x 49 4 x 198 96 x 0.3</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>15 x 6.1 24 x 3 1.52 x 6 7,505 ÷ 5</p> <p>17 x 1½</p> <p>Make links to doubling and halving</p> <p>50 x 28 86 x 50 500 x 70 18 x 2.5</p> <p>86 x 2.5 160 x 35 500 x 88 1.5 x 6.6</p> <p>0.5 x 120 4.5 x 2.2 15% x 346 75% x 220</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>15% x 440 $\frac{2}{5} \times 140$ 24 x 3</p> <p>20% of 1500 95% of 240</p> <p>Multiplying and dividing fractions</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>$\frac{3}{5} \div 3$ $\frac{2}{5} \div 2$ $\frac{3}{4} \div 2$ $\frac{2}{5} \times 140$ $\frac{1}{4} \times \frac{1}{8}$</p>
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Upper KS2 examples (continued)

<p>Re-ordering and finding complements</p> <p>11 + 59 33 + 57 14 + 90 + 86 290 + 310 1.15 + 2.55 0.8 + 0.26</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>1,034 + 586 2.15 + 0.05</p> <p>Circle two numbers that added together make 0.25 0.05 0.23 0.2 0.5</p>	<p>x and ÷ by powers of 10</p> <p>10 x 53 87 x 10 1000 x 14 100 x 8.3 100 x 0.41 30 x 3 7 x 0.3 30 x 30 30 x 70 567 ÷ 100 36 ÷ 10 0.5 ÷ 10 280 ÷ 4</p> <p>5600 ÷ 80 30 = $\square \div 12$ 270 ÷ 9 = $\square \div 0.9$ 7 x 0.001 1.8 ÷ 0.1 3.25 ÷ 0.00001</p> <p>Circle two numbers that multiply together to equal 10 million 200 2,000 5,000 50,000</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>1440 ÷ 12 630 ÷ 9 1,320 ÷ 12 0.9 ÷ 10 20% of 1,800 20% of 1500 7,505 ÷ 5 95% of 240 100 x 412 0.9 ÷ 10 1.28 x 100 50,000 - 500 10 x 100</p> <p>Circle two numbers that multiply together to equal 1 million 200 2,000 5,000 50,000</p>	
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